EXAMINATION II:

Fixed Income Valuation and Analysis

Derivatives Valuation and Analysis

Portfolio Management

Questions

Final Examination

March 2018

Question 1: Fixed Income Valuation and Analysis (50 points)

You are working as an Asset Manager and get the task of setting up a capital guaranteed fund ("Capital Protect") concept using the following securities:

Security/ Feature	Rating	Coupon	Tenor (Maturity)	Yield to maturity (YTM)	Price	Modified Duration	Convexity
Zero-coupon Bond A	AAA	0%	5 years	0.85%	95.86%	①	29.50
Zero-coupon Bond B	AAA	0%	15 years	2.35%	②	14.66	229.11
Zero-coupon Bond C	AAA	0%	35 years	③	37.40%	34.03	④
Equity Fund	n/a	n/a	n/a	n/a	EUR 100	0	0

Notes:

- n/a = Not applicable.
- 1bps = 0.01%.
- Yield Convention: 30/360.
- Zero-coupon Bonds A, B and C are repaid at maturity at par (100%) and are considered default-free. They are quoted in EUR.

a) Before designing such a "Capital Protect" structure you are asked to answer some basic questions:

a1) Calculate the 4 missing values ①, ②, ③ and ④ in the table above. (13 points)

a2) Why is the modified duration of the Equity Fund set as zero? (Give short reasoning, no calculations required.) (4 points)

a3) Using the modified durations data, calculate the approximate yield of an equally weighted portfolio comprising of only Zero Bonds A, B and C.

[Hint: The portfolio yield can be approximated by the weighted average of the given yields, whereby the weights are based on the modified durations.]
[Note: In case you have not answered question a1) use 3% for the yield of Zero Bond C and 5 for the modified duration of Zero Bond A.]
(4 points)

b) You are now confronted with the task to come up with a proposal for 3 "Capital Protect" fund solutions which guarantee the investor a fund value of 100% at maturity while still being able to participate in the upside potential of the equity market:

b1) There are 3 investors X, Y and Z with individual planning horizons in 5, 15 and 35 years and investment amounts of EUR 50,000 each: How much exposure (in EUR) should these 3 investors have in the given Zero Bonds and Equity Fund at inception of the respective funds?

[Hint: Investor X invests only in Zero Bond A and the rest in the given equity fund; Investor Y invests only in Zero Bond B and the rest in the given equity fund; Investor Z invests only in Zero Bond C and the rest in the given equity fund.]
[Note: Use 70% as price for Zero Bond B in case you have not answered question a1).]
(9 points)

b2) Investor X is not satisfied with the low yield of Zero Bond A and suggests to add some credit exposure by investing in a BBB rated 5 year Zero Bond: What problem do you see for the "Capital Protected" fund with such a move? (Give short reasoning, no calculation required.) (4 points)

c) Finally, you are asked to assess the performance of the 3 "Capital Protect" Funds proposed in question b), over a period of one year based on the following scenario:

(i) Parallel increase in interest rates by +150bps and

(ii) Equity Fund performance of +17.5%.

c1) Calculate the holding period rate of return of the 3 "Capital Protect" funds after 1 year in this scenario. Calculate the bonds return with the exact bond price change and don't use the duration approximation, assuming that each of the YTMs of Bond A, B and C increase by 150 bp with respect to the value it had 1 year before. [Note: In case you have not answered question a1) use 3% for the yield of Zero Bond C and 70% as price of Zero Bond B.] (12 points)

	At t0		Bond price		1-year performance		
Investors	Investment in bond	Investment in equity	Bond price in t0	Bond price in t1	Performance bond	Performance equity	1-year return
X							
Y							
Z							

c2) Alternatively, the equity exposure of the "Capital Protect" fund could be generated using equity derivatives (e.g. long call options) instead of the given Equity Fund exposure. Mention 1 key argument in favor of replacing the Equity Fund with long call options and1 key argument against such replacement. (4 points)

Question 2: Derivative Valuation and Analysis **(28 points)**

Answer the following questions about the theoretical price of futures and hedging strategies using futures.

a) You are considering an arbitrage transaction and found an arbitrage opportunity in a futures contract to purchase/sell a non-dividend paying stock (Stock A) at the end of 3 months. The current price of Stock A is USD 1,000, and the current 3-month risk-free interest rate is 2% per annum (continuous compounding). The price of the futures on this Stock A is USD 1,020. Describe the arbitrage transaction by trading one share of the Stock A and calculate the profit that you will earn. Round your calculation to the 1^{st} decimal place.

(5 points)

b) You are thinking about finding the theoretical price of a futures contract to purchase/sell at the end of 6 months a stock (Stock B) with a dividend yield of 1% per annum (continuous compounding). The share price of the Stock B is currently USD 10,000, and the current 6-month risk-free interest rate is 3% per annum (continuous compounding). What is the current theoretical price of the futures on this Stock B? Round your calculation to the 1^{st} decimal place. (4 points)

c) Answer the following questions about stock portfolio strategies using futures.

c1) It is currently January 201N. You want to use an S&P 500 futures maturing in 13 months to hedge the price fluctuations at the end of 1 year for a stock portfolio (Portfolio C) currently priced at USD 20.5 million. The current 1-year risk-free interest rate is 3.5% per annum (continuous compounding), and the dividend yield on the S&P 500 is 1% per annum (continuous compounding). The S&P 500 is currently priced at 2,000, and the S&P 500 futures is traded in units of USD 250 multiplied by the S&P 500 futures price. The stock portfolio C has a beta of 1.5 against the S&P 500. Note that the beta is estimated running a linear regression of the percentage changes in the portfolio value against the percentage changes in the futures price.

Calculate how many units of the S&P 500 futures you should buy or sell. Assume that the market price of the futures is equal to the theoretical price, and round the theoretical price to the 1^{st} decimal place. Indicate the number of contracts as an integer.

(5 points)

c2) One year later in January 201N+1, the S&P 500 is down to 1,800; the 1-year risk-free interest rate is unchanged at 3.5% per annum (continuous compounding), the S&P 500 dividend yield is also unchanged at 1% per annum (continuous compounding) and the stock portfolio still has a beta of 1.5 against the S&P 500.

Calculate the profit/loss on the S&P 500 futures and the profit/loss on the portfolio C over the 1-year period. [Note: in order to calculate the profit/loss on the S&P 500 futures, regardless of c1), assume that in January 201N you traded 60 units of the futures at the theoretical price of 2,055; calculate the theoretical price of the futures in January 201N+1, rounding to the 1st decimal place. In order to calculate the profit/loss on the portfolio, assume that its return is consistent with the CAPM.] (10 points)

c3) First calculate the 1-year rate of return of the hedged portfolio in c2). Then compare this return with the risk-free rate and explain why they are not exactly the same.

(4 points)

Question 3: Derivatives / Derivatives in Portfolio Management (29 points)

As a portfolio manager, you have been assigned to manage a diversified stock portfolio with a present value of CU (Currency Units) 30 billion that has the same structure as the market index, Market Stock Average (MSA). Your job is to manage the portfolio's risk-return by adding futures and options trades while preserving the structure of the stock portfolio.

The current value of the MSA S_0 is CU 20,000, and for simplicity, you can ignore dividends. All of the futures and options traded have the MSA as the underlying asset, mature 3 months from now, and have 1 trading unit worth 1,000 times the MSA. In other words, if the price of 1 unit of an option with a strike price 20,000 has a price of CU x, purchasing 1 trading unit costs CU $1{,}000\,x$. Options are European type, and the risk-free rate r_f is constant at 2% annualized rate. Finally, the current price of a MSA futures is expressed as F_0, the current price of a MSA call option with a strike price of K as C_0, the price of a put option as P_0, and the time to maturity as τ years (i.e. $\tau = 0.25$).

Answer the following questions. Round off your results to the 1st decimal place.

a) Assuming that there are no arbitrage opportunities, write the relationship that holds true between the MSA S_0 and the futures price F_0, and find the theoretical current futures price. (3 points)

b) Assuming that there are no arbitrage opportunities and that futures and options have the same maturity and underlying asset, write the relationship that holds true between the futures price F_0, the call option price C_0, and the put option price P_0 with the same strike price K. (3 points)

c) Currently, the futures price is the same as the theoretical price found in a), and the price of a call option with a strike price K = CU 20,000 is CU 90 higher than the price of a put option (in other words $C_0 - P_0 = 90$). Show an example of an arbitrage transaction using a MSA put option and call option with a strike price of CU 20,000, a MSA futures trade, and borrowing or lending at the risk-free rate, and clearly indicate the profit to be derived therefrom (show the trading units of the futures and options and the type of borrowing or lending in the arbitrage transaction). (8 points)

d) The arbitrage opportunity was immediately resolved, but there are greater expectations of a decline in the MSA. You want to hedge (make the floor for) your portfolio using 3 months put options with a strike price of CU 19,000. Find how many trading units you need to trade in order to hedge your portfolio. (3 points)

e) The current price of the put option in d) is CU 500 per unit. You decide to borrow the purchasing cost of the option at the risk-free rate and repay the value in full in 3 months. Find the floor for the total assets held that will be achieved in 3 months, considering the option cost. Assuming that you establish the floor, draw a graph illustrating the relationship between the value of total asset holdings in 3 months on the vertical axis and the MSA in 3 months on the horizontal axis. (5 points)

f) Instead of using the put option, you want to achieve the floor in e) with a dynamic hedge using MSA futures. Find the position in MSA futures (short or long, number of trading units) that you need to take at the current point in time in order to achieve the floor. Assume that the delta of a MSA put option with a strike price of CU 19,000 is -0.3. (4 points)

g) Immediately after you take the futures position in f), the MSA goes up. How do you need to adjust the futures position to maintain the dynamic hedge and achieve the floor in e)? Describe the directions for position adjustments and explain your reasons. (3 points)

Question 4: Portfolio Management (39 points)

The Onebridge University Foundation has an asset allocation policy in which 50% of assets are invested in stocks and 50% in bonds, both in its home country market. Fig. 1 contains assumptions for expected returns, risks (standard deviations), correlation coefficient for stocks and bonds, and the risk-free asset. Fig. 2 contains an efficient frontier drawn from these assumptions.

At a recent Board meeting, the majority agreed that Foundation should reduce the portfolio risk, even if that would lead to a slight reduction in expected portfolio return. One of the directors, Mr. Bond, pointed out that while the current portfolio appears to allocate equally between stocks and bonds, it is actually extremely biased towards stocks when one considers the risk contributions of stocks and bonds.

To achieve equal risk exposure, Mr. Bond proposes to adopt a risk parity strategy, a strategy that has received much attention recently in the asset management industry. According to Mr. Bond, the expected return of Portfolio R, in which risk is allocated equally between stocks and bonds, is above the fund's minimum target return. A back-testing indicates that this strategy would have outperformed the current fund portfolio from 2000 to the present, particularly during the financial crisis of 2008 in which the risk parity strategy would have reduced the risk.

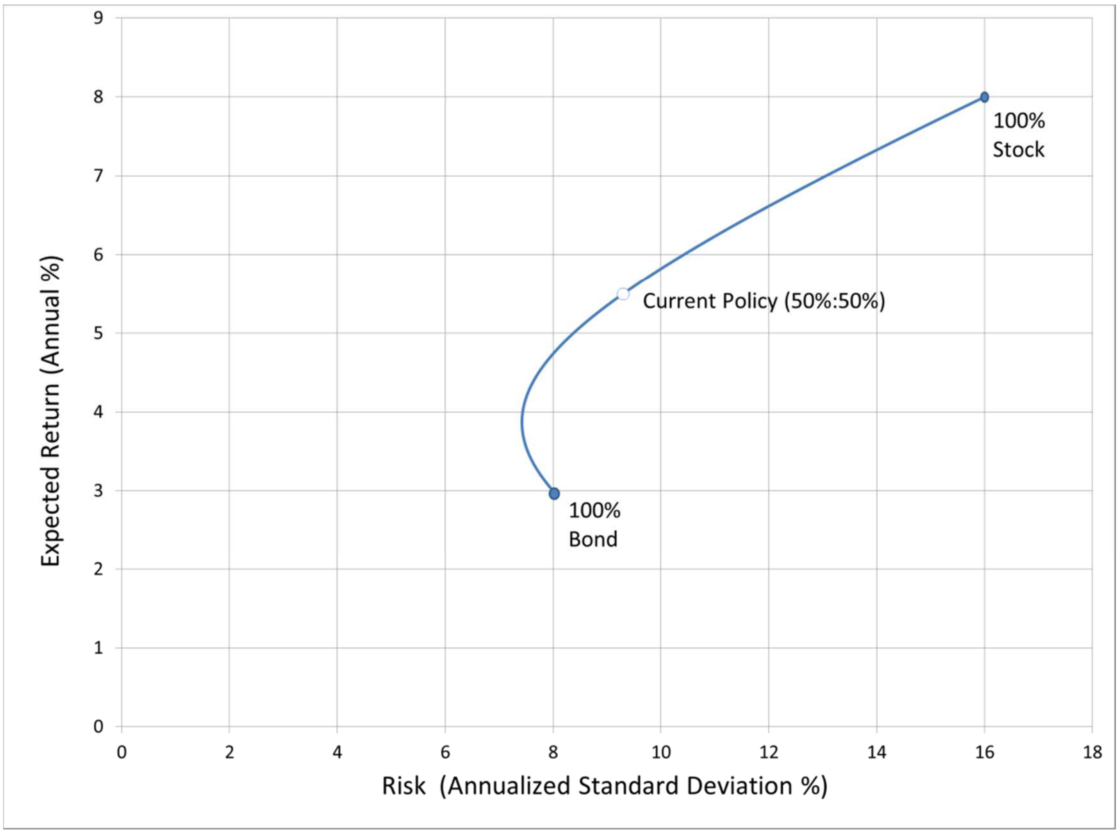
The Board discussed the risk parity strategy, and called in Professor Emeritus Tobin, a noted authority on financial theory, for an advice. Prof. Tobin told them, "You shouldn't forget that there are also risk-free assets. Determining a portfolio of risky assets and determining an allocation between the portfolio and risk-free assets are completely separate issues. First, consider an optimized Portfolio T that is comprised of stocks and bonds, with respective weights of 60% and 40% with a standard deviation of 10.1%. When Portfolio T is combined with risk-free assets, one can find a portfolio P that has the same expected return as the risk parity portfolio R but has a lower risk and a higher Shape ratio than the risk parity portfolio."

Fig. 1: Returns, risks and correlation coefficient by asset class

(Unit: Annualized %)

	Expected return	Risk (standard deviation)
Stocks	8	16
Bonds	3	8
Risk-free asset	2	0
Stock and bond correlation coefficient		0

Fig. 2: Efficient frontier



a) Write the formulas used to determine the price of bonds and stocks based on the DCF (discounted cash flow) approach. Use the formulas to explain how an increase in expected inflation affects bond prices, and how a decline in expected economic growth affects stock prices. (6 points)

b) Calculate the percentage of risk arising from stocks in total risk (i.e., variance) in the portfolio under the current asset allocation (50% in stocks and 50% in bonds). (5 points)

c) Based on the numbers in Fig. 1, what would be the stock and bond allocation weights under a risk parity strategy? What would be the expected return at this time? Express as a percentage (round to two decimal places). In the efficient frontier figure provided above in the introduction, plot the approximate position of Portfolio R.
[Hint: Because stocks and bonds are not correlated, one can equate the variance contributions of stocks and bonds in this question.] (6 points)

d) The risk parity strategy may not perform as well as the back-test results suggest. In what kind of economic environments would the risk parity strategy perform worse than the back-test? In your answer, comment briefly on the effects of monetary policies in developed counties that may have contributed to a strong performance of the risk parity strategy during the past 10 years. (5 points)

e) Regarding Prof. Tobin's comments:

e1) What is the following proposition called: “Determining a portfolio of risky assets and determining an allocation between the portfolio and risk-free assets are completely separate issues.”? (2 points)

e2) Plot Portfolio T (the optimized portfolio suggested by Prof. Tobin) on the efficient frontier figure provided above. What is Portfolio T called, and what properties does it have? (4 points)

e3) Show the allocation weights (of stocks, bonds and risk-free assets) for Portfolio P. (Express your answer in percentages, rounded to 0 decimal places.) Plot Portfolio P on the efficient frontier figure above. (7 points)

e4) Calculate the Sharpe ratio of Portfolio P. (4 points)

Question 5: Portfolio Management **(34 points)**

You are the executive director for investments at Pension Plan A, and you have been trying to invest efficiently on the asset-side so that the pension fund achieves its expected rate of return. However, having seen an convincing argument that pension fund management should consider not only the asset-side but also the liability-side, you decide to incorporate the liability of the pension plan into your analysis. The table below contains current information about asset and liability-sides. Round off all calculations to the 2nd decimal place.

	Initial value (million CUs)	Expected return	Risk (standard deviation)	Correlation against bonds	Correlation against equity
Equity	60	8.00%	15.00%	0.3	1
Bonds	90	3.00%	4.00%	1	0.3
Liability	100	3.50%	6.00%	0.8	0.2

a) Calculate the Pension Plan A's funding ratio. Identify three actions that could be taken by the pension plan in light of the current funding ratio level. (4 points)

b) Calculate the overall expected return and risk (standard deviation) on Pension Plan A assets. (4 points)

c) Calculate the correlation between asset and liability for Pension Plan A. (2 points)

d) Calculate the expected surplus return and surplus risk for Pension Plan A. (4 points)

e) Calculate the required surplus return assuming Pension Plan A's required minimal threshold for the surplus return SP_{min} is -10% and shortfall risk tolerance is 10%. From your results, state whether Pension Plan A is fulfilling its shortfall constraints.
[Hint: Use 1.28 as the 10th percentile for standard normal distribution.] (4 points)

f) To this point, you have studied a surplus approach pension ALM (= Asset and Liability Management) strategy that defines surplus as the market value of liabilities deducted from the market value of assets, and attempts to maintain and manage it. Give an example of another ALM strategy, and explain it briefly. (3 points)

g) The surplus approach controls the difference between assets and liabilities, while the mean-variance approach focuses only on assets and seeks to achieve target returns with minimum risk. In these two approaches, "bonds" and "cash" are positioned differently. The following sentences describe the difference between these two approaches. To complete the sentence, choose the correct word for each blank from the ones in the bracket next to it.

"The mean-variance approach does not consider the pension liability, which means that (1) …………………………[cash/bonds/equity], which has the smallest standard deviation, is the minimum risk asset, and (2) ………………………………[cash/bonds/equity] with (3) …………………………… [interest rate/equity] risk is/are the risky asset. In the surplus approach, as the pension liability and (2) ………………………………[cash/bonds/equity] have high (4) …………………………[correlation/duration], (2) ………………………… [cash/bonds/equity] are the minimum risk asset, but (1) ………………………… [cash/bonds/equity] is the risky asset because it has the (5) ………………………… [duration gap/equity gap] with pension liability".

(1) =

(2) =

(3) =

(4) =

(5) =

(4 points)

h) The surplus approach basically tries to match the durations of assets and liabilities so that interest rate sensitivities are also matched. This means its primary investment focus is bonds. However, many pension funds, including Pension Plan A, also invest in equity. Why? (3 points)

i) Pension Plan A's bond portfolio consists of government bonds issued in developed countries and long and short positions in various futures contracts. The bond portfolio integrates both the regional/country allocation strategy and yield curve strategy. The following table summarize the exposures of the portfolio. For foreign exchange rates, as the fund practices separate overlay management, in your answer you can ignore currency risk/exposures.

	1-3 years	3-7 years	7-15 years	15 years +	Overall
Eurozone	-25	70	200	-10	240
United States	-70	-250	10	30	-260
Overall	-95	-180	210	20	-20

(Modified duration / bps relative to the benchmark)

Answer the following questions.

i1) Comment on the fund's regional and country allocation strategy. How would you describe the fund's outlook for the bond markets in the United States and Eurozone? What would be the rationale behind the fund's allocation strategy? (3 points)

i2) Comment on the yield curve strategy for the United States. How would you describe the fund's market outlook and a possible rationale behind its yield curve strategy?

(3 points)

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Solutions

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Question 1: Fixed Income Valuation and Analysis (50 points)

a)

a1)

The calculations are as follows:

① As it is a zero-coupon bond, Duration$_A$ = Maturity$_A$ = 5y. Then we have:

$$MD_A = \frac{\text{Duration}}{(1+Y_A)} = \frac{5}{(1+0.85\%)} = 4.96$$

② The price of Bond B is: $P_B = \frac{100}{(1+Y_B)^t} = \frac{100}{(1+2.35\%)^{15}} = 70.58\%$

③ We compute the yield to maturity of Bond C (Y_C) as follows:

$$P_C = \frac{100}{(1+Y_C)^t}$$

$$37.4 = \frac{100}{(1+Y_C)^{35}}$$

$$\therefore Y_C = 2.85\%$$

④ The convexity of Bond C is:

$$C_C = \frac{1}{P} \cdot \frac{1}{(1+Y_C)^2} \cdot \frac{t \cdot (t+1) \cdot CF}{(1+Y_C)^t} = \frac{1}{P} \cdot \frac{1}{(1+Y_C)^2} \cdot t \cdot (t+1) \cdot \frac{CF}{(1+Y_C)^t}$$
$$= \frac{1}{P} \cdot \frac{1}{(1+Y_C)^2} \cdot t \cdot (t+1) \cdot P = \frac{1}{(1+Y_C)^2} \cdot t \cdot (t+1) = \frac{1}{(1.0285)^2} \cdot 35 \cdot 36$$
$$= 1{,}191.14$$

[Note that C* = 0.5 · C = 595.57 is an alternative definition of convexity.]

a2)

- The modified duration is a figure that approximates the interest rate sensitivity of interest bearing securities.
- Stocks in the given Equity Fund typically do not exhibit pre-defined behaviors with respect to interest rate movements.
- Depending on the prevailing economic environment, stocks could either rise or fall with rising or falling interest rates.
- As a result, stocks' interest rate sensitivity is often expressed by a zero modified duration.

a3)
[Note: The YTM is a complex average of spot rates. In a portfolio context, it has been proven, that in a normal interest rate environment, weighting the YTM by the modified duration gives a better approximation of the exact portfolio YTM.]

The yield (YTM_P) of the equally weighted zero bond portfolio is calculated as the modified duration weighted average of the bond yields:

$$YTM_P = \frac{(MD_A \cdot YTM_A + MD_B \cdot YTM_B + MD_C \cdot YTM_C)}{(MD_A + MD_B + MD_C)}$$

$$= \frac{4.96 \cdot 0.85\% + 14.66 \cdot 2.35\% + 34.03 \cdot 2.85\%}{4.96 + 14.66 + 34.03} = 2.53\%$$

Alternatively (using given values):

$$YTM_P = \frac{(MD_A \cdot Y_A + MD_B \cdot Y_B + MD_C \cdot Y_C)}{(MD_A + MD_B + MD_C)}$$

$$= \frac{5 \cdot 0.85\% + 14.66 \cdot 2.35\% + 34.03 \cdot 3\%}{5 + 14.66 + 34.03} = 2.62\%$$

b)
b1)
The Zero Bond and Equity exposures (in EUR) of the 3 given investors can be derived using the formula: $\text{Zero bond exposure} = \text{Price of Zero bond} \cdot 50{,}000$. As the investors must get EUR 50,000 at maturity even if the equity part is zero, they will invest 50,000 notional in the zero-coupon bonds (which will be repaid at par at maturity).

- Investor X:

$\text{Zero bond exposure}_A = 95.86\% \cdot 50{,}000 = 47{,}930$

$\text{Equity Fund exposure}_A = 50{,}000 - 47{,}930 = 2{,}070$

- Investor Y:

$\text{Zero bond exposure}_B = 70.58\% \cdot 50{,}000 = 35{,}290$

$\text{Equity Fund exposure}_B = 50{,}000 - 35{,}290 = 14{,}710$

[Alternatively (using given value):

$\text{Zero bond exposure}_B = 70\% \cdot 50{,}000 = 35{,}000$

$\text{Equity Fund exposure}_B = 50{,}000 - 35{,}000 = 15{,}000$

- Investor Z:

$\text{Zero bond exposure}_B = 37.40\% \cdot 50{,}000 = 18{,}700$

$\text{Equity Fund exposure}_B = 50{,}000 - 18{,}700 = 31{,}300$

b2)
The problem with taking a lower credit exposure can be expressed as follows:

- The "Capital Protect" concept is based on a Zero Bond redemption of 100% at the respective maturity.
- In other words: The capital protection or guarantee is ensured by the 100% redemption price of the underlying Zero Bond.
- The default probability of AAA rated Zero Bonds can be neglected even over longer periods.
- However, when it comes to investing in a BBB rated Zero Bond the default-free assumption, and hence the protection cannot be maintained any more.

c)
c1)
The funds' performance after 1 year in the given scenario can be calculated as follows:

Zero bond prices after parallel increase in interest rate by +150bps:

$P_{A,1} = \frac{100}{(1+2.35\%)^4} = 91.13 \Rightarrow$ Price change $= 91.13 - 95.86 = -4.73$, or in %, we have price change is 91.13 / 95.86-1 = -4.93%.

$P_{B,1} = \frac{100}{(1+3.85\%)^{14}} = 58.93 \Rightarrow$ Price change $= 58.93 - 70.58 = -11.65$, or in %, we have price change is 58.93 / 70.58-1 = -16.51%.

$P_{C,1} = \frac{100}{(1+4.35\%)^{34}} = 23.51 \Rightarrow$ Price change $= 23.51 - 37.4 = -13.89$, or in %, we have price change is 23.51 / 37.4-1 = -37.14%.

Hence the performances can be calculated as follows:

	At t=0		Bond price		1-year performance		
Investors	**Investment in Bond**	**Investment in Equity**	**Bond price in t0**	**Bond price in t1**	**Performance bond**	**Performance equity**	**1-year return**
X	95.86%	4.14%	95.86	91.13	-4.93%	17.50%	**-4.01%**
Y	70.58%	29.42%	70.58	58.93	-16.51%	17.50%	**-6.50%**
Z	37.40%	62.60%	37.40	23.51	-37.14%	17.50%	**-2.94%**

Alternatively (using given values):

$P_{B,1} = \frac{100}{(1+3.85\%)^{14}} = 58.93 \Rightarrow$ Price change $= 58.93 - 70 = -11.07$, or in %, we have price change is 58.93 / 70-1 = -15.81%

$P_{C,1} = \frac{100}{(1+4.5\%)^{4}} = 22.39 \Rightarrow$ Price change $= 22.39 - 37.4 = -15.01$, or in %, we have price change is 22.39 / 37.40-1 = -40.13%

Alternative

	At t=0		Bond price		1-year performance		
Investors	**Investment in Bond**	**Investment in Equity**	**Bond price in t0**	**Bond price in t1**	**Performance bond**	**Performance equity**	**1-year return**
X	95.86%	4.14%	95.86	91.13	-4.93%	17.50%	**-4.01%**
Y	70.00%	30.00%	70.00	58.93	-15.81%	17.50%	**-5.82%**
Z	37.40%	62.60%	37.40	22.39	-40.13%	17.50%	**-4.06%**

c2)

Argument for and against use of derivatives:

- (FOR) Long call option positions allow for higher equity exposure (e.g. via 'cheaper' call options instead of underlying stocks), creating a leverage, thus resulting in better fund performance with rising equity markets (protection by Zero Bonds is not impacted).
- (AGAINST) Long call option positions have higher equity exposure, creating a leverage, and therefore perform less well in falling equity markets (protection by Zero Bonds is not impacted).
- (AGAINST) Long call option positions could involve counterparty risk vis-à-vis for instance the seller of the equity call options.

[NB: if derivative position is collateralized or against central clearing counterparty one could argue that whilst there is no counterparty risk any more in downturn scenario call options might expire worthless while equity position still has residual value; Equity Future positions have such residual value as well but require initial and variation margins in terms of cash which reduces the Zero Bond and, hence the protection value.]

Question 2: Derivative Valuation and Analysis (28 points)

a)

Since the theoretical futures price, $F = 1{,}000 \cdot \exp^{(2\% \cdot 3/12)} = \text{USD } 1{,}005$, is lower than the market price of the futures, you buy the underlying spot and sell the futures.

At t = 0 you borrow USD 1,000 at the riskless interest rate for 3 months; you purchase 1 share, and you sell one futures contract.

At t = 3 months, you must return $F = 1{,}000 \cdot \exp^{(2\% \cdot 3/12)} = \text{USD } 1{,}005$; you obtain USD 1,020 from the futures trade in exchange for the share you hold. By subtraction, you earn a certain profit of 1,020 – 1,005 = USD 15 in 3 months.

b)

$$F = S \cdot e^{(r-y)\cdot t} = 10{,}000 \cdot e^{(3\% - 1\%)\cdot 0.5} = 10{,}100.5$$

c)

c1)

The current theoretical price of the S&P 500 futures is $2{,}000 \cdot \exp^{(3.5\% - 1\%)\cdot 13/12} = 2{,}054.9$

The price of 1 unit of futures is $2{,}054.9 \cdot 250 = 513{,}725$

Therefore, $N_f = -\dfrac{1.5 \cdot 20.5 \text{ million}}{513{,}725} = -59.86$, so you sell 60 contracts.

[Additional explanation:

In this case, the beta is estimated as shown below.

$$\frac{\Delta S}{S} = \alpha + \beta \cdot \frac{\Delta F}{F} + \varepsilon$$

The beta expresses the change in the rate of change of the spot price resulting from a change in the rate of change of the futures price. Therefore:

$$\frac{\frac{\Delta S}{S}}{\frac{\Delta F}{F}} = \frac{F}{S} \cdot \frac{\Delta S}{\Delta F}$$

This means that the following relationship holds true between the hedge ratio and the beta:

$$HR \equiv \frac{\Delta S}{\Delta F} = \beta \cdot \frac{S}{F}$$

The number of contracts required for the hedge is found from the following relationship between the futures trading units and the number of underlying assets:

$$N_f = -HR \cdot \frac{N_s}{K}$$

Therefore:

$N_f = -\beta \cdot \dfrac{S}{F} \cdot \dfrac{N_s}{K}$. Note that $S \cdot N_s$ expresses the value of the portfolio.]

c2)
At the end of 1 year, the S&P 500 is at 1,800. Note that the price of futures against the index maturing in 1 month will be $1{,}800\cdot\exp(3.5\%-1\%)\cdot(1/12)=1{,}803.8$

Therefore, the profit from the futures is $60\cdot(2{,}055-1{,}803.8)\cdot 250=\text{USD }3{,}768{,}000$

The index return, continuously compounded, is: $r=\ln(1{,}800/2{,}000)=-10.536\%$
So, the expected rate of return on the portfolio according to the CAPM is:
$3.5\%+1.5\cdot(-10.536\%+1\%-3.5\%)=-16.054\%$

Therefore, the loss on the portfolio is: $20.5\text{ million}\cdot\left[e^{-16.054\%}-1\right]=-3{,}040{,}496$

This produces a total profit of $3{,}768{,}000-3{,}040{,}496 = 727{,}503$

c3)

The rate of return on the hedged portfolio is $\dfrac{727{,}503}{20{,}500{,}000}=3.54\%$, whereas the rate of return on a riskless asset is 3.5%.

In theory, a perfectly hedged portfolio should have a return equal to the risk-free rate. In the current case there is a small difference, due to the rounding of the number of contracts.

Question 3: Derivatives / Derivatives in Portfolio Management (29 points)

a)
Dividends and transaction costs can be ignored, so from the no-arbitrage condition, the MSA futures price F_0 is given as:

$$F_0 = S_0 \cdot (1 + r_f)^{\tau} = 20,000 \cdot (1.02)^{0.25} = 20,099.3$$

[Or $F_0 = S_0 \cdot e^{r_f \cdot \tau} = 20,000 \cdot e^{2\% \cdot 0.25} = 20,100.3$]

b)
From the above equation and the put-call parity:

$$C_0 - P_0 = S_0 - \frac{K}{(1+r_f)^{\tau}} \Leftrightarrow C_0 - P_0 = \frac{S_0 \cdot (1+r_f)^{\tau} - K}{(1+r_f)^{\tau}} \Leftrightarrow C_0 - P_0 = \frac{F_0 - K}{(1+r_f)^{\tau}}$$

(in this equation, $\tau = 0.25$)

[Or, $F_0 = K + (C_0 - P_0) \cdot (1+r)^{\tau}$]

c)
It should be: $C_0 - P_0 = S_0 - \frac{K}{(1+r_f)^{\tau}} = 20,000 \cdot \left[1 - \frac{1}{(1.02)^{0.25}}\right] = 98.7685$. Since 90 < 98.7685, the call is undervalued, or the put is overvalued, and there is an arbitrage opportunity:

At t = 0:

- Long 1 Call option with K = 20,000
- Short 1 Put option with K = 20,000
- Short 1 futures
- Borrow 1,000 · (C_0 - P_0)

Payoff in 3 months:
S_T expresses the value of the MSA index at maturity in 3 months.

- Payoff from the long call and short put = S_T - K
- Payoff from the futures = F_0 - S_T.
- Borrowing at the risk-free rate: repayment of 1,005 · (C_0 - P_0) = 1,005 · 90

This trade has a balance of zero at the current point in time, and will earn a certain profit in 3 months, as shown below:

$$1,000 \cdot (S_T - K) + 1,000 \cdot (F_0 - S_T) - 1,005 \cdot 90 = 99,300 - 90,450 = 8,850$$

Alternatively:

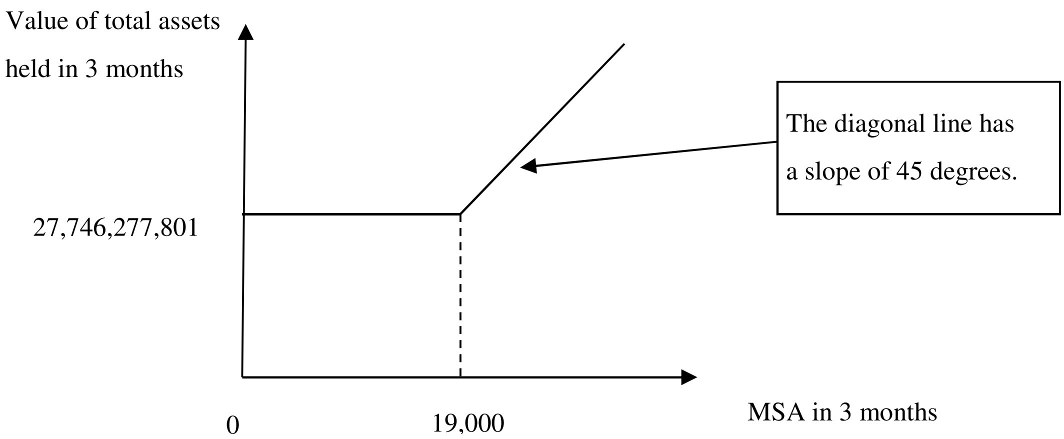
Position	Initial Cost$_{t=0}$	Final Value$_{T=0.25}$[Note:K=20,000]	
		S_T<K	S_T>K
Buy bond [= invest money]	-1,000·[20,000/1.02$^{0.25}$]	+1,000·K	+1,000·K
Buy Call	-1,000 · [90]	0	+1,000· (S_T−K)
Sell Put		-1,000 · (K−S_T)	0
Sell short Index	+1,000 · [20,000]	-1,000 · S_T	-1,000 · S_T
TOTAL	+1,000 · [8.7685]	**0**	**0**

d)
For the total assets held to be constant at a certain level when the put option is in-the-money, it is necessary to take a position in which the loss on the stock portfolio is fully offset by the return on the put option if, in 3 months, the MSA is below the strike price of CU 19,000. The value of the stock portfolio is CU 30 billion, which is 1.5 million times the MSA, and the option is traded in units of 1,000. Therefore, 1500 trading units of the option should be purchased.

In summary: 30 billion / (20,000 · 1,000) = 1,500 put options.

e)
The floor for total assets held is found by subtracting the repayment of the option purchasing price in 3 months from the floor achieved by the put option. Therefore, the floor will be:
$19,000 \cdot 1,000 \cdot 1,500 - 500 \cdot 1,000 \cdot 1,500 \cdot (1.02^{0.25}) = 27,746,277,801$

The following graph shows the relationship between the total assets held and MSA in 3 months.



f)

The futures delta is $\frac{\partial F_0}{\partial S_0} = 1.02^{0.25} \cong 1.005$, and the delta of a put option with a strike price of CU 19,000 is -0.3. For a dynamic hedge, it is necessary to take a futures position so that the delta of a position purchasing 1 trading unit of the put option is equal to the delta of the futures position. Therefore, it is necessary to take a position of $-\frac{0.3}{1.005} = -0.299$ trading units in the futures per unit put option (0.299 trading unit short position).

In this case, to create the floor in d), 1,500 trading units of the put option are purchased, so the dynamic hedge requires $1{,}500 \cdot (-0.299) = -448$ trading units of the MSA futures. In other words, the futures position to be taken at the current point in time to achieve the dynamic hedge is "a short position of 448 trading units."

g)

If all other conditions are equal and the MSA (the underlying asset) rises, the delta of the put option will rise. In other words, it will be a negative but smaller absolute value (i.e. from -0.30 to -0.20 for example). Therefore, to maintain the dynamic hedge, it is necessary to reduce the futures short position (change the delta of the futures position to match the delta of the put option position that will achieve the floor), or buy back the futures.

Question 4: Portfolio Management (39 points)

a)
Bond inflation risk:

Expressing coupon as c, principal as F, yield as r, and the timing of future payments as t (maturity as T), the bond price P_{Bond} is:

$$P_{Bond} = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} + \frac{F_T}{(1+r)^T}$$

Generally, the interest rate r (denominator discount rate) rises as the inflation rate rises, so the bond price P will fall.

Stock low growth risk:

Expressing dividends as D, dividend growth rate as g, risk-free rate as r_f, and the stock risk premium as λ, and by using constant-growth dividend discount model (Gordon model), the share price P_{Stock} is determined as follows:

$$P_{Stock} = \frac{D}{(r_f + \lambda) - g}$$

Under lower economic growth, r will most likely decline, but λ will generally rise, g will fall, and the denominator as a whole will most likely rise. On the other hand, corporate earnings will suffer, and D will most likely decline. As a result, share price P declines.

b)
The current total risk of the portfolio (variance) is:

$$\sigma_P^2 = w_E^2 \cdot \sigma_E^2 + w_B^2 \cdot \sigma_B^2 + 2 \cdot w_E \cdot w_B \cdot \sigma_E \cdot \sigma_B \cdot \rho_{E,B}$$

$$= (0.5 \cdot 16\%)^2 + (0.5 \cdot 8\%)^2 + 2 \cdot 0.5 \cdot 0.5 \cdot 16\% \cdot 8\% \cdot 0 = 80$$

The covariance in the third term is 0, and the variance from stocks is 64 in the first term.

Stock's contribution to total risk is $\frac{64}{80} = 80\%$

c)
Since, $w_E + w_B = 1$, we have $w_B = 1 - w_E$

The first two terms in the decomposition formula for variance shown in b) are equal to the risk allocations to stocks and bonds below:

$$w_E^2 \cdot (16\%)^2 = (1 - w_E)^2 \cdot (8\%)^2$$

$$\therefore w_E = 33\% \text{ and } w_B = 67\%$$

Therefore, expected return of the portfolio is:

$$33\% \cdot 8\% + 67\% \cdot 3\% = 4.67\%$$

d) The economic environment has deteriorated since 2000 with the collapse of the IT bubble, the collapse of Lehman Brothers, and the European fiscal crisis. The central banks in developed countries therefore adopted easy money policies that dropped bond interest rates to historically low levels. A risk parity strategy with higher allocation to bonds performed well historically because of the decline in bond interest rates under monetary easing, and because of repeated stock market crashes. If the economy recovers and inflation rates rise, interest rates will rise and bond prices fall, leading to concerns that the risk parity strategy may not perform as well as back testing would indicate.

e)

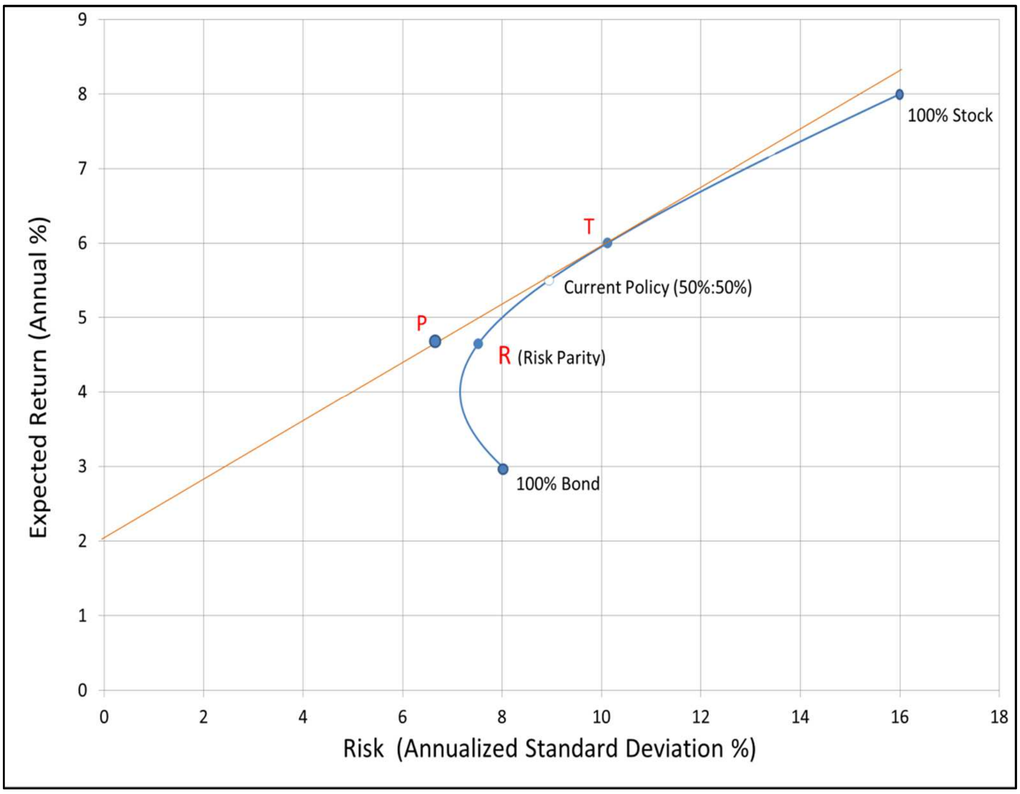
e1)

It is called “Separation theorem”.

e2)

T is called the “tangent portfolio”

The tangent portfolio is the point of maximum slope for straight lines drawn from the risk-free interest rate to points on the frontier.



e3)

The expected return of Portfolio T is $60\% \cdot 8\% + 40\% \cdot 3\% = 6\%$

The expected return of Risk Parity Portfolio R is 4.67%, so the following holds true:

If the weight of P allocated to risk-free assets is w, then $w \cdot 2\% + (1-w) \cdot 6\% = 4.67\%$

Therefore, solving we get the weight of risk-free assets as 33%:

Stocks are $60\% \cdot (1-w) = 60\% \cdot 67\% \approx 40\%$

Bonds are $40\% \cdot (1-w) = 40\% \cdot 67\% \approx 27\%$

e4)
The Sharpe ratio is the same for P and T. Measuring at T:

$$\frac{6\% - 2\%}{10.1\%} = 0.396$$

Question 5: Portfolio Management (34 points)

a)

$$\text{Funding ratio } (\text{FR}) = \frac{\text{Value of asset}}{\text{Value of liability}}$$

$$= \frac{60+90}{100} = 150\%$$

Three actions the pension plan could take:

- Currently, the funding ratio is 150%, and the plan is overfunded.
- It is possible to reduce the surplus, and the pension plan could take any of the following three actions to do so:
  (1) Contribution holiday.
  (2) Raise benefit levels.
  (3) Sell a part of assets and allow the pension plan sponsor to withdraw the excess funding.

b)

Overall expected return on assets:

$$\mu_A = W_E \cdot \mu_E + W_B \cdot \mu_B$$
$$= (60/150) \cdot 8.0\% + (90/150) \cdot 3.0\%$$
$$= 5.00\%$$

Overall risk on assets:

$$\sigma_A = \sqrt{(W_E \cdot \sigma_E)^2 + 2 \cdot W_E \cdot W_B \cdot \sigma_E \cdot \sigma_B \cdot \rho_{EB} + (W_B \cdot \sigma_B)^2}$$
$$= \sqrt{(0.4 \cdot 15\%)^2 + 2 \cdot 0.4 \cdot 0.6 \cdot 15\% \cdot 4\% \cdot 0.3 + (0.6 \cdot 4\%)^2}$$
$$= 7.1\%$$

c)

Correlation between asset and liability:

$$\rho_{AL} = \frac{W_E \cdot \sigma_E \cdot \rho_{LE} + W_B \cdot \sigma_B \cdot \rho_{LB}}{\sigma_A}$$
$$= \frac{0.4 \cdot 15\% \cdot 0.2 + 0.6 \cdot 4\% \cdot 0.8}{7.1\%} = 0.44$$

d)

Expected surplus return:

$$\mu_{SP} = FR \cdot \mu_A - \mu_L$$
$$= 150\% \cdot 5\% - 3.5\% = 4\%$$

Surplus risk:

$$\sigma_{SP}=\sqrt{\left(FR\cdot\sigma_A\right)^2-2\cdot\left(FR\cdot\sigma_A\right)\cdot\sigma_L\cdot\rho_{AL}+\left(\sigma_L\right)^2}$$

$$=\sqrt{\left(150\%\cdot 7.1\%\right)^2-2\cdot\left(150\%\cdot 7.1\%\right)\cdot 6\%\cdot 0.44+\left(6\%\right)^2}$$

$$= 9.66\%$$

e)

Required surplus return:

$$SP_{min}+Z_{\alpha}\cdot\sigma_{SP}=-10\%+1.28\cdot 9.66\%=2.36\%$$

Pension Plan A's expected surplus return: $\mu_{SP}=4\%$

Pension Plan A's expected surplus return (4.0%) is above the required surplus return (2.36%). It therefore satisfies the shortfall constraint.

f)

- Simulation type: Use such as Monte Carlo simulation to determine, within the scope of risk tolerance, an optimum policy asset mix from the efficient frontier so that the assets match liability trends.
- Cash flow matching type: Also called "dedication," cover benefit payments, which are liabilities; with bond coupon income and redemption at maturity so that cash flows from liabilities and assets more or less match.

g)

"The mean-variance approach does not consider the pension liability, which means that (1) cash, which has the smallest standard deviation, is the minimum risk asset, and (2) bonds with (3) interest rate risk are the risky asset. In the surplus approach, as the pension liability and (2) bonds have high (4) correlation, (2) bonds are the minimum risk asset, but (1) cash is the risky asset because it has the (5) duration gap".

(1) cash
(2) bonds
(3) interest rate
(4) correlation
(5) duration gap

h)

- In current markets, bond expected returns are low and it is difficult to achieve required returns just by investing in bonds. It is therefore common practice for funds to allocate a portion of their portfolio to equities, which are high-risk, high-return assets.
- Equities are also thought to serve as a hedge against inflation over the long term.
- Because pension sponsors want to reduce pension contributions by raising expected returns on pension assets

i)
i1)
Duration strategy: Eurozone long/United States short

Market outlook: Comparing bond markets in the United States and Eurozone, the fund expects the Eurozone bond market to rise (declining interest rates), and the US bond market to fall (rising interest rates).

Reasons: Duration measures the sensitivity of bond values to interest rate changes. Relative modified duration for the United States is negative -260 bps, in preparation for future interest rate rises. Meanwhile, Eurozone relative modified duration is positive +240 bps, so the position seeks return from declining interest rates.

i2)
Yield curve strategy: For the United States, the yield curve strategy is negative (short) for short-term/medium-term modified duration, but positive (long) for long-term modified duration. This is a flattening strategy.

Market outlook: The fund expects the FED to raise policy interest rates, which will cause a rise in interest rates in the short and medium-term zones, but it does not expect much of a rise (or decline) in long-term interest rates.