EXAMINATION II:

Fixed Income Valuation and Analysis

Derivatives Valuation and Analysis

Portfolio Management

Questions

Final Examination

September 2018

Question 1: Fixed Income Valuation and Analysis **(51 points)**

As an analyst in the Fixed Income area, you are working with the following bonds (annual coupons on a "30/360" basis).

Table 1

Type of Bond / Features	Time to Maturity	Rating	Coupon	Price	Yield to Maturity (YTM) excl. credit spread *	Modified Duration	Convexity
Government Bond	5 years	AAA	0.00%	①	−0.30%	④	⑥
Corporate Bond	3 years	BBB	0.75%	②	0.00%	2.97	⑦
Covered Bond	1 year	AA	0.25%	③	−0.10%	⑤	2.00

* This column represents the YTM of the bonds EXCLUDING credit spread.
[YTM = $YTM_{excluded\_credit\_spread}$ + credit spread]

On top of the above given yields to maturity excluding credit spread, the market is charging the following credit spreads, which are based uniquely on rating and time (therefore you have identical credit spreads for government, corporate and covered bonds which have the same rating and the same time to maturity; 1bps = 0.01%):

Table 2: Credit spreads based on rating and time

Rating / Time to maturity	1 year	2 years	3 years	4 years	5 years
AAA	0 bps	0 bps	0 bps	0 bps	0 bps
AA	5 bps	10 bps	15 bps	25 bps	35 bps
A	10 bps	20 bps	35 bps	45 bps	55 bps
BBB	25 bps	35 bps	40 bps	70 bps	90 bps

a) Calculate the missing values ① − ⑦ in the first table above. (22 points)

b) Calculate the modified duration and the convexity of a portfolio comprising EUR 30 million in the given government bond, EUR 20 million in the corporate bond and EUR 40 million in the covered bond.

[Note: In case you have not fully answered question a) assume 5 & 30 as the modified duration & convexity of the government bond, 12 as the convexity of the corporate bond and 1 as the modified duration of the covered bond.] (6 points)

c) Calculate the approximate average yield to maturity of a portfolio based on the given EUR-volumes from question b) and the modified durations of the three bonds.

[Hint: Practice shows that the true average YTM of a portfolio is better approximated using not only the weights but also the modified durations of each individual bond composing the portfolio.]

[Note: In case you have not fully answered question a), assume modified durations for the government bond and covered bond of 5 and 1 respectively.] (5 points)

d) What is the holding period return after 1 year when investing in the given corporate bond assuming a rating uplift to single A and a yield for 2 years to maturity excluding credit spread of minus 0.05% (-0.05%)? [Assume that the credit spreads based on rating and time contained in Table 2 remain unchanged.] (7 points)

e) Explain why the prices of some premium bonds with the passing of time are decreasing in value despite a decrease in their yield to maturity. (No calculations required.) (6 points)

f) Calculate the "YTM including credit spread" of a putable Corporate Bond with the same characteristics as the bond in question d) assuming an option premium of 10bps p.a. (i.e. time to maturity: 2 years; rating: A; YTM excluding credit spread: −0.05%). (5 points)

Question 2: Derivative Valuation and Analysis **(17 points)**

On January 1st, the current market price of a stock that does not pay dividends is USD 50, and the rate of return on a riskless asset for a period of 1 year is 3% p.a. continuously compounded. After 6 months, on July 1st, the market price of the stock has risen to USD 55. At this point in time, the rate of return on a riskless asset for a period of 6 months is also 3% p.a. continuously compounded.

a) Calculate the theoretical price (on January 1st) of a forward contract to purchase this stock in 1 year. (3 points)

b) What would the value (profit/loss) of the forward contract be at the time it is first entered into? (2 points)

c) After six months, on July 1st, what would be the value (profit/loss) of the forward contract which was analysed 6 months earlier? (4 points)

d) There is a European-type call option on the stock currently priced at USD 50 with a strike price of USD 47 which expires in 1 year. This option has a market price of USD 2 at the current point in time. You sense an arbitrage opportunity in the price of the call option. Describe the trading strategy to exploit the arbitrage opportunity, and calculate the profit to be earned from the arbitrage transaction [Note: no put available on this stock]. (8 points)

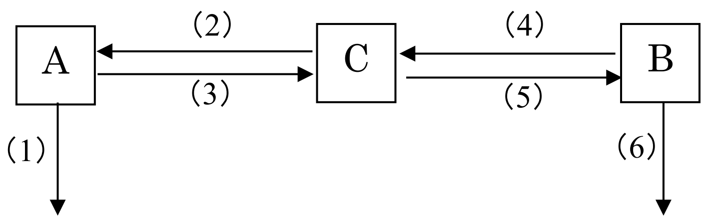
Question 3: Derivative Valuation and Analysis (17 points)

Company A and Company B, are multinational corporations that have many dealings with companies in Japan and the United States. They also do business with Bank C, which has presented them with a proposal for a fixed lending rate (annualized rate for 2 years). Interest payments occur annually. The current spot foreign-exchange rate is 100 JPY/USD (1 USD = 100 JPY).

	Lending rate in USD	Lending rate in JPY
Company A	7.0%	5.0%
Company B	6.0%	3.5%

a) Explain the comparative advantages of Company A raising funds in USD and Company B raising funds in JPY. (4 points)

b) Company A raises USD 100 million for 2 years and Company B raises JPY 10 billion for 2 years. At the same time, Company A considers executing a currency swap with Bank C that would switch its borrowing to JPY 10 billion for 2 years. Likewise, Company B considers a currency swap with Bank C that would switch its borrowing to USD 100 million for 2 years. Bank C asks for 20 basis points (annualized rate) as fees and takes all foreign-exchange risks. Conversely, Company A and Company B are considering currency swaps in which they take no foreign-exchange risks and equally split the total remaining profit from the swap. In the figure below, write the currencies and amounts of payments 1 year after execution of the currency swap in blanks (1) to (6).



(1) =

(2) =

(3) =

(4) =

(5) =

(6) =

(13 points)

Question 4: Derivatives / Derivatives in Portfolio Management (42 points)

You are a fund manager in charge of a diversified stock portfolio with the same composition as the Market Stock Average (MSA) and with a present value of CU 10 billion (CU = Currency Unit). Your job is to manage the portfolio's risk and enhance the returns by adding futures and options while maintaining its composition.

At the current point in time (t = 0), the MSA average stock price index is 10,000 and the risk-free rate is 4% (annualized – simple rate). The market also trades futures and European options with the MSA as the underlying asset.

Assume for simplicity that dividends may be ignored. Also assume that all futures and options mature in 3 months from now and that 1 trading unit (contract) is 1000 times the MSA. (In other words, if the price of 1 unit of an option with a strike price 10,000 is CU C, the cost of purchasing 1 trading unit is CU 1000 C.)

If there is a need to express in symbols, use r_f for the risk-free interest rate (annualized), F_0 for the current futures price of MSA futures, S_0 for the current value of the INDEX, $C_0(K)$ for the current price of an MSA call option with strike price K, $P_0(K)$ for the price of a put option, and τ (years) for the time to maturity.

a) There are growing concerns about a decline in the stock market. The price of an MSA put option with a strike price 10,000 is higher than the price of a call option with the same 10,000 strike price, and the difference between them (the price for 1 unit of the put option minus the price for 1 unit of the call option) is CU 10. However, the price of MSA futures is equivalent to the theoretical price, and funds can be borrowed at the risk-free rate. You have found an arbitrage opportunity.

a1) Find the theoretical price of the MSA futures. (Show the equation used to derive it.) (3 points)

a2) Is the put/call parity satisfied? Explain. (4 points)

a3) Describe the transactions needed at t = 0 and in 3 months to realise an arbitrage transaction. The number of put option contracts to be used is 1. Detail the cashflows occurring at t = 0 and in 3 months, and calculate the arbitrage profit earned in 3 months. (9 points)

b) The arbitrage opportunity described in a) immediately disappeared, and concerns about a fall in the MSA grew even stronger. You decide to use futures and options to create a hedge against the risk of a decline in the value of assets under management.

b1) First, you decide to trade the futures contract to reduce exposure to the MSA. You want the beta of your portfolio with respect to the MSA to be at the level of 0.5. Find the position in the MSA futures (short or long, the number of trading units) required to achieve the target beta. (4 points)

Concerns about a decline grow even stronger, and you decide to forgo the futures trade in b1) and instead create a floor that maintains the value of assets under management 3 months from now at or above a certain level. To do this, you consider using the MSA put option shown below.

Strike price K	Option price $P_0(K)$	Delta
8,000	209	-0.145

b2) Find the put option position (short or long, the number of trading units) needed to establish the floor. Take the position so that the overall value of assets under management (including the option payoff) will be constant if the option goes in-the-money. (4 points)

b3) Draw a graph placing the overall value of assets under management achieved by maintaining the put option position that creates the floor in b2) for 3 months on the vertical axis, and the value of the MSA in 3 months on the horizontal axis. Also find the value of the floor. Assume that borrowings are used to fund the purchase of options and are repaid in 3 months. (5 points)

b4) Right before you purchase the put options, there is news that raises the volatility of the MSA. What will be the impact on the floor value found in b3)? Explain your reasons. (No calculations required.) (3 points)

c) You consider using MSA futures to achieve a dynamic hedge instead of trading put options.

c1) Find the MSA futures position (short or long, number of trading units) you should take initially to achieve the floor calculated in b3) with dynamic hedging. (Round the intermediate calculations to the 3rd decimal place.) (5 points)

c2) How should you change the futures position in response to price changes in the MSA to maintain the dynamic hedge? Explain. (No calculations required.) (5 points)

Question 5: Portfolio Management **(33 points)**

Your team is preparing a financial and operational report to provide a comprehensive look at the performance of two investment funds, the *Argon European Stock Fund* (fund *A*) and the *Cobalt European Mutual Fund*, (fund *C*), by applying the CAPM. Both funds are denominated in EUR and have the EURO STOXX 50 (market *M*) as benchmark.

a) Based on your careful analysis, you estimate the expected returns for the next year as $\overline{R}_A = 6.1\%$ and $\overline{R}_C = 8.5\%$, while the corresponding sensitivities of each fund with respect to the benchmark are given by $\beta_A = 0.92$ and $\beta_C = 1.15$.

   Knowing that the 12 month Euribor rate is $r_F = -0.1\%$ and the expected return for the market index is $\overline{R}_M = 7.2\%$, calculate the expected equilibrium return for each fund. (4 points)

b) Assume that the expected risks of the two investment funds for next year (in percentage) are respectively $\sigma_A = 4.5\%$ and $\sigma_C = 5.8\%$. Calculate the Sharpe Ratio for each fund and then comment on the result in terms of risk/return tradeoff. (6 points)

c) Plot a chart of the Security Market Line (SML) and indicate the two mutual funds in the chart. (4 points)

d) Compare the expected return and the theoretical return of the two funds. Based on your analysis, how would you evaluate the two funds? Discuss carefully. (5 points)

Now, you focus on the opportunity of protecting a managed portfolio, whose current value is EUR 120 million. The portfolio is well diversified, so that the major portfolio risk comes from the market risk. You plan to do this by using put options on the EURO STOXX 50 index (the contract size is equal to EUR 10), where the beta of the stock portfolio with respect to the index is equal to 1.15 and the option maturity is exactly six months from now. The target is to constitute a floor to insure your portfolio against a decline in capital value (after paying out dividends) of more than 7.5% over the next semester (six months). Assumed that the dividend yield of the portfolio is 0.9% per six months, while the dividend yield on the EURO STOXX 50 index is 0.6% per six months. Now the risk-free interest rate is zero and the last quotation of the EURO STOXX 50 is 3450.

e) Applying the CAPM rule and assuming that the insurance cost is to be borne externally from the managed funds, calculate the theoretical strike price to achieve the protective put strategy described above. (9 points)

f) How many EURO STOXX 50 option contracts should you hold to implement the protective put strategy? (5 points)

Question 6: Portfolio Management **(20 points)**

Mr. Jones has been hired as a financial advisor for a small investment firm. He has identified two Exchange-Traded Funds (ETFs) to include in a portfolio for a new client. The two funds are expected to perform differently depending on the economic environment according to the following parameters.

State of the economy	Probability of the state	Return Maverick ETF	Return Bear ETF
Normal	80%	15%	5%
Recession	20%	2%	10%

a) Calculate the individual expected returns for Maverick ETF and Bear ETF. (4 points)

b) Mr. Jones's new client has USD 175,000 to invest in these two funds of which USD 61,250 will be allocated to the Bear ETF, and the remainder to the Maverick ETF. Calculate the expected return of this portfolio. (4 points)

c) Calculate the standard deviation of the portfolio described in question b). (8 points)

d) Assume the client has a target return of 8%. What percentage should be allocated to each ETF given the goal is an expected return of 8%? (4 points)

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Solutions

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Question 1: Fixed Income Valuation and Analysis (51 points)

a)

Consider the government zero coupon bond: YTM $= -\ 0.3\% + 0 = -\ 0.3\%$

$$P_0 = \frac{100}{\left(1-0.3\%\right)^5} = 101.51 \qquad ①$$

$$D^{Mod} = \frac{D}{1+y} = \frac{5}{1-0.3\%} = 5.02 \qquad ④$$

$$C = \frac{1}{P} \cdot \frac{1}{(1+Y)^2} \cdot \frac{t_i(t_i+1) \cdot CF_i}{\left(1+Y\right)^{t_i}}$$

$$= \frac{1}{101.51} \cdot \frac{1}{(1-0.3\%)^2} \cdot \frac{5 \cdot 6 \cdot 100}{\left(1-0.3\%\right)^5} = 30.18 \qquad ⑥$$

Consider the corporate bond: YTM $= 0 + 0.40\% = 0.4\%$

$$P_0 = \sum_{i=1}^{N} \frac{CF_i}{\left(1+R_i\right)^{t_i}} = \frac{0.75}{\left(1+0.4\%\right)} + \frac{0.75}{\left(1+0.4\%\right)^2} + \frac{100.75}{\left(1+0.4\%\right)^3} = 101.04 \qquad ②$$

$$C = \frac{1}{\sum_{i=1}^{N} \frac{CF_i}{\left(1+Y\right)^{t_i}}} \cdot \frac{1}{(1+Y)^2} \cdot \sum_{i=1}^{N} \frac{t_i(t_i+1) \cdot CF_i}{\left(1+Y\right)^{t_i}}$$

$$= \frac{1}{101.04} \cdot \frac{1}{(1+0.4\%)^2} \cdot \left(\frac{1 \cdot 2 \cdot 0.75}{\left(1+0.4\%\right)^1} + \frac{2 \cdot 3 \cdot 0.75}{\left(1+0.4\%\right)^2} + \frac{3 \cdot 4 \cdot 100.75}{\left(1+0.4\%\right)^3} \right) \qquad ⑦$$

$$= 11.79$$

Consider the covered bond: YTM $= -\ 0.1\% + 0.05\% = -\ 0.05\%$

$$P_0 = \frac{100.25}{\left(1-0.05\%\right)} = 100.3 \qquad ③$$

$$D^{Mod} = \frac{D}{1+y} = \frac{1}{1-0.05\%} = 1.001 \qquad ⑤$$

b)

The portfolio duration is calculated as:

$$D_P^{Mod} = \sum_{i=1}^{N} x_i \cdot D_i^{Mod} = \frac{30}{90} \cdot 5.02 + \frac{20}{90} \cdot 2.97 + \frac{40}{90} \cdot 1.001 = 2.78$$

The portfolio convexity is calculated as:

$$\text{Portfolio convexity} = \sum_{i=1}^{N} w_i \cdot C_i = \frac{30}{90} \cdot 30.18 + \frac{20}{90} \cdot 11.79 + \frac{40}{90} \cdot 2 = 13.57$$

Using given values, the solution would be:

$$D_P^{Mod} = \sum_{i=1}^{N} x_i \cdot D_i^{Mod} = \frac{30}{90} \cdot 5 + \frac{20}{90} \cdot 2.97 + \frac{40}{90} \cdot 1 = 2.77$$

$$\text{Portfolio convexity} = \sum_{i=1}^{N} w_i \cdot C_i = \frac{30}{90} \cdot 30 + \frac{20}{90} \cdot 12 + \frac{40}{90} \cdot 2 = 13.56$$

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c)

The portfolio yield is given by:

$$YTM_P \cong \sum_{j=1}^{N} \left(\frac{PV_j \cdot D_j^{mod}}{\sum_{i=1}^{N} PV_i \cdot D_i^{mod}} \right) \cdot YTM_j$$

$$= \frac{30 \cdot 5.02 \cdot (-0.3\%) + 20 \cdot 2.97 \cdot 0.4\% + 40 \cdot 1.001 \cdot (-0.05\%)}{30 \cdot 5.02 + 20 \cdot 2.97 + 40 \cdot 1.001}$$

$$= -0.09\%$$

Using given values, the solution would be:

$$YTM_P \cong \sum_{j=1}^{N} \left(\frac{PV_j \cdot D_j^{mod}}{\sum_{i=1}^{N} PV_i \cdot D_i^{mod}} \right) \cdot YTM_j$$

$$= \frac{30 \cdot 5 \cdot (-0.3\%) + 20 \cdot 2.97 \cdot 0.4\% + 40 \cdot 1 \cdot (-0.05\%)}{30 \cdot 5 + 20 \cdot 2.97 + 40 \cdot 1} = -0.09\%$$

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d)

Price of the corporate bond after 1 year, with $YTM = -0.05\% + 0.2\% = 0.15\%$:

$$P_1 = \frac{0.75}{(1+0.15\%)} + \frac{100.75}{(1+0.15\%)^2} = 101.2$$

$$HPR_{0,1} = \frac{101.2 + 0.75 - 101.04}{101.04} = 0.9\%$$

e)

Two effects determine the value development of a bond over time:

1. "Roll down/up the yield curve"
2. "Pull to par"

In case of some premium bonds the "Pull to par" when approaching maturity is dominating the "Roll down/up the yield curve" effect. As a result, these bonds lose value despite decreasing YTMs.

f)

We know that price of puttable bond = price of straight bond + price of put option. So the price of a puttable bond is always higher than the price of a straight bond because the put option adds value to an investor. Therefore the yield of a puttable bond is always **lower** than the yield of a straight bond.

We have:

YTM including credit spread = YTM without credit spread + credit spread - premium
= -0.05% + 0.20% - 0.10% = +0.05%.

Question 2: Derivative Valuation and Analysis (17 points)

a)
The theoretical price of the forward contract is:
$F_{t,T} = S_t \cdot e^{r(T-t)} = 50 \cdot e^{3\% \cdot 1} = \text{ USD } 51.52$

b)
The value of the forward contract at the time it is first entered into is 0.

c)
In order to solve the question, you have to find the 6-month forward and then to calculate the profit.

The theoretical price of the forward contract is:
$F_{t,T} = S_t \cdot e^{r(T-t)} = 55 \cdot e^{3\% \cdot 0.5} = \text{ USD } 55.83$

The current value of the forward contract purchased 6 months earlier is:
$(55.83 - 51.52) \cdot e^{-3\% \cdot 0.5} = 55 - 51.52 \cdot e^{-3\% \cdot 0.5} = 55 - 50.75 = \text{USD } 4.25$

d)
The lower limit of the theoretical price for this European-type call option is:
$\text{Min}\left[C_E(S,\tau,K)\right] = S_t - Ke^{-r \cdot \tau} = 50 - 47e^{-3\%} = \text{USD } 4.39$
However, the market price is 2 dollars, so there is an arbitrage opportunity.

If, at the current point in time, we short the stock and purchase the call option, we obtain $50 - 2 = \text{USD } 48$, which is invested in a riskless asset for 12 months to be $48 \cdot e^{3\%} = \text{USD } 49.46$.

If, after 12 months, the share price is at or above USD 47, we exercise the call option, use the stock acquired to close the short position, and earn $49.46 - 47 = \text{USD } 2.46$.

Conversely, if the share price is less than 47 dollars, you do not exercise the call option, but purchase stock on the market to close the short position. As a result, you earn $49.46 - S_t > \text{USD } 2.46$.

Question 3: Derivative Valuation and Analysis (17 points)

a)
Company A needs to pay $7-6=1\%$ more in USD and $5-3.5=1.5\%$ more in JPY than Company B would. The lower interest-rate gap in US dollars means that Company A has a comparative advantage in raising US dollars funds. In contrast, Company B has comparative advantage in raising funds in Japanese yen.

b)
The total profit for the entire transaction is 50 basis points $\left((500-350)-(700-600) \ =150-100=50\right)$. Bank C receives 20 basis points, so the remainder $\left(50-20/2 \ =15\text{ bp}\right)$ is split between Company A and Company B. Company A pays a fixed interest of $(5.0-0.15) \ =4.85\%$ in JPY and Company B pays a fixed interest of $(6.0-0.15) \ =5.85\%$ in USD.

Therefore, in the figure below:

(1) USD 100 million · 7% = USD 7 million;

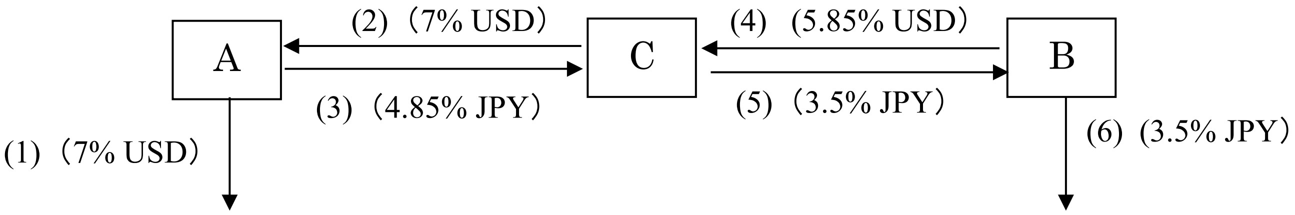
(2) USD 7 million ;

(3) JPY 10 billion · 4.85% = JPY 485 million;

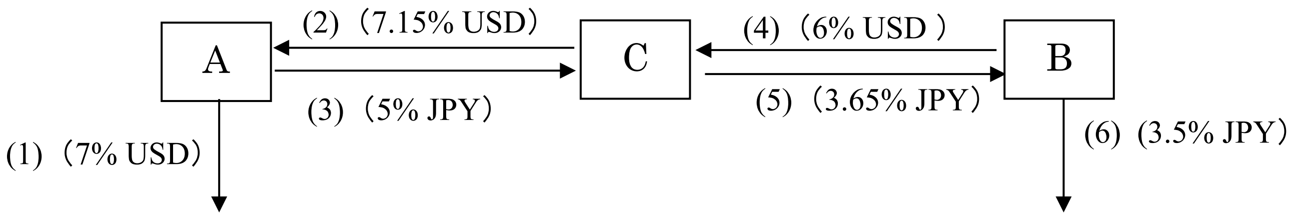
(4) USD 100 million · 5.85% = USD 5.85 million;

(5) JPY 10 billion · 3.5% = JPY 350 million;

(6) JPY 350 million .



Note: the following answer is also correct:



Question 4: Derivatives / Derivatives in Portfolio Management (42 points)

a)

a1)

Dividends and transaction costs can be ignored, so the following can be used to derive the MSA futures average from the no-arbitrage condition.

$$F_{t,T} = S_t \cdot \left(1 + r\left(T\text{-}t\right)\right) = 10{,}000 \cdot \left(1 + 4\% \cdot 0.25\right) = 10{,}100$$

a2)

$C_0 - P_0 = -10$. On the other side, $S_0 - \dfrac{K}{1 + r_f \cdot \tau} = 10{,}000 - \dfrac{10{,}000}{1 + 4\% \cdot 0.25} \cong 99.01$.

-10 ≠ 99, therefore, the Put-Call Parity does not hold true. Arbitrage is possible.

a3)

As $C_0 - P_0 = -10$ but should be equal to +99 (see question a2), this means that the put option is too expensive by CU 109 (= 99 - (-10)).

So, the arbitrage will consist of selling the expensive put option, and buy a synthetic put, built with Put-Call parity:
C – P = S – K / (1 + rt) → P = C – S + K / (1 + rt)

To trade S, we would rather use the futures contract, whose price we calculated at 10,100 (see question a1), and lend or borrow the net flow of all the trades.

Initiation (all flows to be multiplied by quotity = 1,000):

Trade	**Money Flow (x 1,000)**
Sell Put	+X
Buy Call	-Y
We know that +X-Y =	+10
Sell Futures (no flow with a futures contract)	0
Initial Cash Flow	**+10**
Flow lent at 4% (cf. question statement)	-10
Net Initial Cash Flow	**0**

Three months later (all flows to be multiplied by quotity = 1,000):

	S < 10,000	**S > 10,000**
Short Put	-(10,000 – S)	0
Long Call	0	S – 10,000
Short Futures	10,100 – S	10,100 – S
Lending	10 * (1 + 4% * 0.25)	10 * (1 + 4% * 0.25)
	110.10	**110.10**

Hence, the arbitrage profit is 110.10 x 1,000 = 110,100 CU

b)
b1)
Assets under Management = CU 10 billion with a beta of 1.0 ("…same composition as the index").
Target exposure = beta 0.50 = 10 billion x 0.5 = CU 5 billion
Hence, we need to sell CU 5 billion.

Using futures priced at 10,100 (see question a1), with a size of 1,000 times the index:
-5 billion / (10,100 x 1000) = -495

Hence, we need to sell short 495 futures contracts.

[Or,

$$N_F = -HR \cdot \frac{\text{Market value of spot position}}{\text{Futures contract size} \cdot \text{Spot asset price}}$$

$$HR = \beta \cdot \frac{S_t}{F_{t,T}}$$

HR = 0.5 x 10,000 / 10,100 = 0.495
N_F = -0.495 x 10 billion / (1,000 x 10,000) = -495 = sell short 495 futures contracts.

Alternatively,
The delta of the futures contract ignoring dividends = (1 + 0.04 / 4) / (1 + 0) = 1.01.
We need to short futures to reduce the delta by 0.5. To do so, short:
0.5 / 1.01 = 0.49505 ≈ 0.495 units of the futures contract per unit MSA,
or 0.495 x 1,000 = 495 futures contracts.]

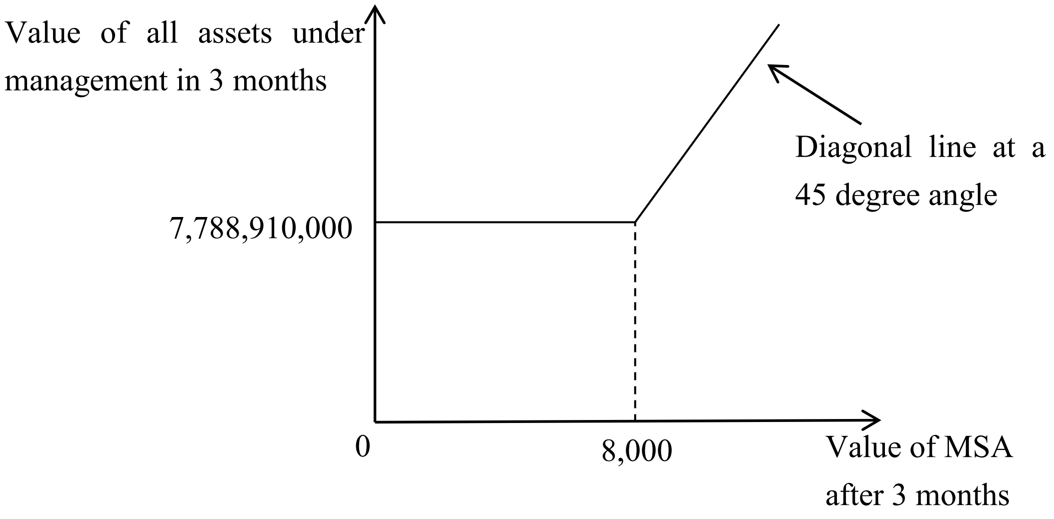
b2)
For the overall value of assets under management (including put options) to remain constant if the put option goes in-the-money, it is necessary to take a position in which the losses on the stock portfolio are offset by the returns on the put option if the MSA is below the strike price, 8,000, in 3 months. The value of the stock portfolio is CU 10 billion, which is 1 million times the MSA, and 1 trading unit of the option is 1,000 contracts, so you should purchase $1{,}000{,}000 / 1{,}000 = 1{,}000$ trading units.

[Or,
Portfolio Size / (Spot price of Index x Option Size)
= 10 billion / (10,000 x 1,000) = 1,000 put options to be purchased.]

b3)
The answer is to subtract the future value of the option premium in 3 months from the strike price of the put option. The floor that is achieved is therefore:

$$8{,}000 \cdot 1{,}000{,}000 - 209 \cdot 1{,}000{,}000 \cdot (1 + 4\% \cdot 0.25) = \text{CU } 7{,}788.91 \text{ million}$$

The relationship between the overall value of the assets under management and the value of the MSA in 3 months is shown in the graph below:



b4)
An increase in the volatility of the underlying asset, the MSA, will increase the price of the put option. This will increase the cost of purchasing options to create the floor and will reduce the value of the floor achieved.

c)
c1)
Because the futures price:
$F_0 = S_0 \cdot (1 + r_f \cdot \tau) = 10,000 \cdot (1 + 4\% \cdot 0.25) = 10,000 \cdot 1.01$
the futures delta is $\frac{\partial F_0}{\partial S_0} = 1.01$.

Meanwhile, the delta of a put option with a strike price of 8,000 is -0.145. For a dynamic hedge, it is necessary to establish a futures position so that the delta of a position purchasing 1 trading unit put options is equivalent to the delta of the futures position. It is therefore necessary to take a position of futures $\frac{-0.145}{1.01} = -0.143$ trading units (short position of 0.143 trading units) per trading unit put options.

In addition, to create the floor in b), you purchase 1,000 trading units of put options and need 1,000 times this for the dynamic hedge. The futures position that you should take at this time to achieve the dynamic hedge is therefore "a short position of 143 trading units."

[Or:
Put Delta = -0.145
Delta in CU = Units x Size x Spot Price x Delta = 1000 x 1000 x 10,000 x -0.145 = -1.45 billion
Replicating this delta with futures contracts, with price = 10,100 and size = 1000:
1.45 billion / (10,100 x 1000) = -143 futures contracts to be shorted.]

c2)
This dynamic hedge is a trading strategy that synthesizes the payoff of the put option by continuously adjusting the futures position to maintain the same delta as the put option, which varies according to movements of the MSA.

All other conditions being equal, a rise (decline) in the value of the underlying asset, the MSA, will result in an increase (decrease) in the delta of the put option. (The absolute value of the negative delta will become smaller (larger).) Therefore, you purchase (sell more) futures so that the delta of the futures position matches the new option delta, and the absolute value of the short position becomes smaller (larger).

Question 5: Portfolio Management (33 points)

a)

The expected equilibrium return derives from the CAPM formula:

$$\overline{R}_{P,CAPM} = r_f + \frac{\overline{R}_M - r_f}{\sigma_M} \cdot \sigma_P = r_f + \left(\overline{R}_M - r_f\right) \cdot \beta_P$$

Therefore, we have:

$$\overline{R}_{A,CAPM} = -0.1\% + \left(7.2\% - \left(-0.1\%\right)\right) \cdot 0.92 = 6.616\%$$

$$\overline{R}_{C,CAPM} = -0.1\% + \left(7.2\% - \left(-0.1\%\right)\right) \cdot 1.15 = 8.295\%$$

b)

$$\text{Sharpe ratio} = \frac{\overline{R}_P - \overline{r}_f}{\sigma_P}$$

Therefore, we have:

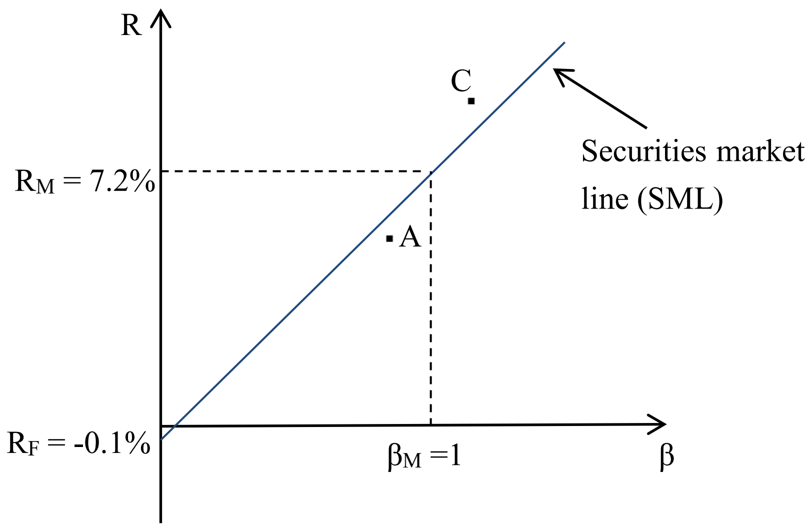
$$\text{Sharpe ratio}_A = \frac{\overline{R}_A - \overline{r}_f}{\sigma_A} = \frac{6.1\% - \left(-0.1\%\right)}{4.5\%} = 1.378$$

$$\text{Sharpe ratio}_C = \frac{\overline{R}_C - \overline{r}_f}{\sigma_C} = \frac{8.5\% - \left(-0.1\%\right)}{5.8\%} = 1.483$$

The Sharpe ratio measures the premium per unit of risk in absolute terms, that is to say that it does not measure the performance of the funds compared to the assigned benchmark EURO STOXX 50. By comparing the two funds only through this measure, fund C is preferred to fund A, because with the same risk unit, there is a higher return.

c)

Below the Security Market Line graph with the position of the two mutual funds:



d)
Comparing the expected return and the theoretical return of the two funds, we obtain that:

$$\overline{R}_{A,CAPM} = 6.616\% > 6.1\% = \overline{R}_A$$

$$\overline{R}_{C,CAPM} = 8.295\% < 8.5\% = \overline{R}_C$$

Alternatively, we have:

$$\alpha_A = \overline{R}_A - \overline{R}_{A,CAPM} < 0$$

$$\alpha_C = \overline{R}_C - \overline{R}_{C,CAPM} > 0$$

where α_P denotes the *Jensens's α* for the security *P*:

$$\alpha_P = \left(\overline{R}_P - r_F\right) - \left(\overline{R}_M - r_F\right) \cdot \beta_P = \overline{R}_P - \overline{R}_{P,CAPM}$$

Hence, using the CAPM, we can say that the *Argon European Stock Fund* (fund *A*) is overpriced $\left(\alpha_A < 0\right)$, while the *Cobalt European Mutual Fund*, (fund *C*) is underpriced $\left(\alpha_C > 0\right)$. This suggests selling your investment in fund Argon, and purchase instead the Cobalt fund.

e)
Let us compute the variation in the EURO STOXX 50 over the next semester, say $r_{MC,0.5}$, corresponding to a 7.5% decline in the capital value of the investment portfolio, that is $r_{PC,0.5} = -7.5\%$.

From the *Security Market Line* equation characterizing the CAPM model (as described on the above question a) we have:

$$r_P = r_F + \beta \cdot \left(r_M - r_f\right)$$

$$r_{PC} + r_{PD} = r_F + \beta \cdot \left(r_{MC} + r_{MD} - r_f\right)$$

and, keeping in mind that our target is over the next six months (not per annum!):

$$r_{PC,0.5} + \frac{r_{PD}}{2} = \frac{r_F}{2} + \beta \cdot \left(r_{MC,0.5} + \frac{r_{MD}}{2} - \frac{r_f}{2}\right)$$

Now, let's compute the capital index return $r_{MC,0.5}$

$$r_{MC,0.5} = \frac{1}{\beta} \cdot \left(r_{PC,0.5} + \frac{r_{PD}}{2}\right) - \frac{1-\beta}{\beta} \cdot \frac{r_F}{2} - \frac{r_{MD}}{2}$$

$$= \frac{1}{1.15} \cdot \left(-7.5\% + 0.9\%\right) - 0.6\% = -6.339\%$$

Therefore, a 7.5% drop in the capital value over the next semester corresponds to a 6.34% drop in the EURO STOXX 50 index. It can be observed that the result is consistent with the beta level and the dividend yield proportion.

It follows that the theoretical strike price of the EURO STOXX 50 put option which defines the above protective strategy is equal to the *floor*, says Φ, applied to the benchmark (not to the managed portfolio!):

$$\Phi = 3450 \cdot \left(1 + r_{MC,0.5}\right) = 3450 \cdot \left(1 - 6.34\%\right) \cong 3231$$

[Alternative answer:

$$\Phi = 3450 \cdot e^{r_{MC,0.5}} = 3450 \cdot e^{-6.34\%} \cong 3238$$]

f)
Finally, we just have to calculate the number of EURO STOXX 50 put options to buy (long position!).

Denoting N_{PUT} as the number of put options with the theoretical strike price $\Phi = 3238$ to buy, we have

$$N_{PUT} = \beta \cdot \frac{\text{Portfolio Value}}{\text{Index Level} \cdot \text{Option Contract Size}}$$

$$= 1.15 \cdot \frac{120 \cdot 10^6}{3450 \cdot 10} = 4000$$

Question 6: Portfolio Management (20 points)

a)
Expected return of the two ETFs are calculated as:

$$E(R_{Maverick}) = \sum_{k=1}^{K} p_k \cdot x_k = 0.8 \cdot 15\% + 0.2 \cdot 2\% = 12.4\%$$

$$E(R_{Bear}) = \sum_{k=1}^{K} p_k \cdot x_k = 0.8 \cdot 5\% + 0.2 \cdot 10\% = 6.0\%$$

b)
The weights of the two ETFs are:

$$w_{Baer} = \frac{61,250}{175,000} = 35\%$$

$$w_{Maverick} = 1 - w_{Baer} = 65\%$$

Expected return of the portfolio is calculated as:

$$\begin{aligned} E(R_P) &= w_{Maverick} \cdot E(R_{Maverick}) + w_{Bear} \cdot E(R_{Bear}) \\ &= 0.65 \cdot 12.4\% + 0.35 \cdot 6\% = 10.16\% \end{aligned}$$

c)
In order to calculate the standard deviation of the portfolio let us first calculate the expected portfolio returns under the two economic scenarios:

In a normal year:

$$\begin{aligned} E(R_P) &= w_{Maverick} \cdot E(R_{Maverick}) + w_{Bear} \cdot E(R_{Bear}) \\ &= 0.65 \cdot 15\% + 0.35 \cdot 5\% = 11.5\% \end{aligned}$$

In a recession year:

$$\begin{aligned} E(R_P) &= w_{Maverick} \cdot E(R_{Maverick}) + w_{Bear} \cdot E(R_{Bear}) \\ &= 0.65 \cdot 2\% + 0.35 \cdot 10\% = 4.8\% \end{aligned}$$

Standard deviation of the portfolio is calculated as follows:

$$Var(X) = \sigma_X^2 = E\left[(X - E(X))^2\right] = E(X^2) - E(X)^2 = \sum_{k=1}^{K} p_k (x_k - E(X))^2$$

$$\begin{aligned} \sigma_P &= \sqrt{\sum_{k=1}^{K} p_k \cdot (x_k - E(X))^2} \\ &= \sqrt{0.8 \cdot (11.5\% - 10.16\%)^2 + 0.2 \cdot (4.8\% - 10.16\%)^2} = 2.68\% \end{aligned}$$

d)

If the expected portfolio return is 8%, we have:

$$E(R_P) = w_{Maverick} \cdot E(R_{Maverick}) + w_{Bear} \cdot E(R_{Bear})$$

$$8\% = w_{Maverick} \cdot 12.4\% + (1 - w_{Maverick}) \cdot 6\%$$

$$\therefore w_{Maverick} = 31.25\%$$

and $w_{Bear} = 68.75\%$