Certified International Investment Analyst

EXAMINATION II:

Fixed Income Valuation and Analysis

Derivatives Valuation and Analysis

Portfolio Management

Questions

Final Examination

September 2019

Question 1: Fixed Income Valuation and Analysis **(28 points)**

Assume that you are responsible for managing a small pension fund. Pension liabilities feature a modified duration of 12.00 and a convexity of 200.

You are supposed to invest in the three following securities:

Table 1: Available Bonds

Security	Coupon	Maturity	Yield to Maturity	Price	D_{Mod}	Convexity
Government Bond	0.00%	30 years	1.75%	①	②	③
Covered Bond	2.00%	10 years	2.50%	95.62%	8.92	73
Corporate Bond	4.00%	7 years	3.50%	103.06%	6.04	36

Note: D_{Mod} = modified duration; 30/360 and annual coupon payments; "short selling" is not possible

a) Before setting up your pension fund's asset-liability-matching-strategy, determine the missing values ①, ② and ③ in the table above. (4 points)

b) Determine (approximately) the absolute price change of the government bond in case of a 125bp interest rate instant increase. Rely on the D_{Mod} as well as the convexity you determined in question a) for obtaining a solution.

[Note: If you didn't answer question a), assume a price of 60.00%, a D_{Mod} of 29.5 and a convexity of 900.] (4 points)

c) Now determine the approximative price of the government bond after a 125bp interest rate increase by only relying on the D_{Mod}. Compare your result to the one you obtained in question b) and to the exact new price. Which approximative price is more precise, the one in question b) or question c)?

[Note: If you didn't answer question b), assume an absolute price change of −18.00.]

(6 points)

d) You decide to hedge your portfolio against interest rate volatility by means of matching modified duration and convexity of assets and pension liabilities. What percentage do you need to invest in the three different securities in order to hedge your fund against a parallel shift of the yield curve? [Hint: You might want to start the analysis by setting up three equations to determine weight, modified duration and convexity of your portfolio consisting of the three securities.]

[Note: If you didn't answer question a), assume a price of 60.00%, a D_{Mod} of 29.5 and a convexity of 900.] (10 points)

e) Which problems could arise in case the immunization strategy is exclusively based on the modified duration (i.e. you do not consider convexity)? How many solutions to the

equations in d) do you have? What happens if the interest rate change is large? Explain briefly, no calculation required. (4 points)

Question 2: Fixed Income Valuation and Analysis **(16 points)**

While analysing the market, you spot an inflation-linked government bond (so-called linker) where the nominal is linked to inflation. The bond has a fixed annual coupon of 2.00% (real), a nominal value of 100 and a term-to-maturity (TTM) of just three years. The coupon has just been paid and the consumer price index (CPI) climbed to currently 110 from 100 upon issuance.

a) Determine the (nominal) purchase price in case you buy this linker at a real yield of 1.00%, and a CPI of 110. (3 points)

b) A year after your purchase in question a), the consumer price index has increased by 1.50% and you decide to sell the linker at a real yield of 1.00%. Determine the nominal holding period return (in %) for the year you held the linker. (3 points)

c) Currently, the real yield for linkers with a term-to-maturity of 3 years stands at 1.00% and the yield-to-maturity for a standard government bond (bullet, with nominal coupon and value) with a three-year maturity is 2.00%. Determine the consumer price index's implied annual growth rate (i.e. Expected Inflation Rate) over the next three years. (3 points)

d) How would the consumer price index's growth rate need to develop so that the nominal yield of the linker would be higher than the yield of a standard government bond? (3 points)

e) The rate you determined in question b) must not necessarily correspond to the consumer price index's growth rate the market expects. Identify two reasons why this could be the case and explain whether or not these will cause the yield of a linker to increase or decrease. (4 points)

Question 3: Fixed Income Valuation and Analysis **(15 points)**

As a fixed income analyst at a renowned investment bank, you have started to look at two different securities, a five-year convertible which is in general also called "exchangeable bond", and a five-year bullet bond.

The five-year convertible bond issued by MunichRe Finance features the following characteristics: The bond has an annual coupon rate of 2.00%, a "AA" rating (S&P) and a face value of EUR 10,000. It is convertible into SAP shares, [different from the bond–issuing company], at a conversion ratio of 44.5514 shares. The quoted price of the convertible bond stands at 102.16%.

The second security is a five-year bullet bond, also issued by MunichRe Finance. It carries a coupon of 3.00% p.a., has a "AA" rating (S&P) and a face value of EUR 10,000. The quoted price of the bullet bond stands at 94.4741% and the yield-to-maturity is at 4.25%.

Further assume that the company SAP, which has a "A" rating from S&P, has shares that are actively traded. Assume that the current price of those shares stands at EUR 162.00 and that their dividend yield is 0.90%.

a) Calculate the conversion price of SAP shares. (2 points)

b) Calculate the conversion premium. Debate the valuation of the convertible bond from the traditional premium analysis point of view.

   [Hint: Assume that the cheapness of the convertible bond is determined by the level of conversion premium, i.e. the conversion premium less than 20% implies the bond is cheap.] (5 points)

c) Calculate the convertible bond's straight value (or pure debt value), which can be calculated by applying the yield to maturity of the similar straight bond to this convertible bond. (3 points)

d) Determine and discuss the convertible bond's payback period.

   [Note: if you did not calculate the conversion premium in c), assume it is equal to 40%.] (3 points)

e) Would the yield to maturity of the convertible have been higher or lower, if the issuer of the convertible had been SAP instead of Munich Re? Explain. No calculations required. (2 points)

Question 4: Derivative Valuation and Analysis and Derivatives in PM (33 points)

You are a portfolio manager assigned to manage a diversified stock portfolio with a present value of JPY 40 billion and the same composition as the Nikkei Stock Average index. Your responsibility is to add futures and options trading to manage the portfolio's risks and returns while still maintaining its structure. The current value of the Nikkei Stock Average index, S_0, is JPY 20,000 and the simple annual risk-free rate is 4%. Answer the following questions. Round the numbers to the 1st decimal place.

For simplicity, assume that dividends and transaction costs can be ignored. All traded futures and options on the Nikkei index have a maturity of 3 months from now. All options are of European type. For the options and futures 1 trading unit is 1,000 times the Nikkei. (In other words, the cost of purchasing 1 trading unit of futures and options is 1,000 times the price of the futures or option). The current price of the Nikkei index futures is F_0. The current prices of call and put options on the Nikkei index with strike price K and τ years to maturity are denoted by $C_0(K)$ and $P_0(K)$, respectively.

a) Show the no-arbitrage relation between the Nikkei index price S_0 and the futures price F_0. Calculate the numerical value of the current futures price. (4 points)

b) Assume that the Nikkei index futures contract trades at the price F_0 calculated in question a). The prices of call and put options with common strike K = JPY 20,000 are $C_0(K)$=JPY 897 and $P_0(K)$ = JPY 678, respectively. By using the put-call parity relation, explain why there is an arbitrage opportunity. Also provide an example of an arbitrage transaction using the futures, options, borrowing, and/or lending. Show the number of contracts traded. Calculate the profit from the arbitrage transaction.

   [Note: If you didn't answer question a), assume a theoretical value of the futures price of JPY 20,100.] (7 points)

c) The arbitrage opportunity in question b) is immediately resolved, but there is a greater potential for a large rise or fall in the Nikkei index in the near future. You decide to bet on this possibility and trade call and put options with strike price K = JPY 20,000 to obtain a profit. What kind of positions should you take in those options? Describe a specific trading strategy and draw its payoff diagram at maturity. What is the maximum loss at maturity?

   Assume that the options can be sold short, and funds can be borrowed or lent at the risk-free rate. (6 points)

d) The possibility described in question c) disappeared as well. You are now very concerned that the Nikkei index will fall over the next 3 months. To reduce the risk of a fall, you decide to take a position in Nikkei index put options with strike price JPY 19,000. Show the put option position (whether a long or short position, and how many trading units) that you should take to create a floor of a certain value for total assets (total value of all asset holdings, including stock portfolio, futures, options and risk-free assets) if put options are in-the-money at the end of 3 months. (3 points)

e) Calculate the floor created at the end of 3 months for the position derived in question d) if the current price of the put option is JPY 320 per unit. Draw a graph of the relationship between total assets held and the Nikkei index, placing total value of assets held at the end of 3 months on the vertical axis and the value of Nikkei index at the end of 3 months on the horizontal axis.

Assume that the purchasing costs for the options (resp. revenues from sale) at the current point in time can be raised by borrowing (resp. lending) at the risk-free rate and repaying (resp. collecting) the entire amount in 3 months.
[Note: If you didn't answer question d), assume a position of 2,010 put options.] (5 points)

f) You decide to achieve the floor in question e) with a dynamic hedge using Nikkei index futures instead of the put option.

Calculate the initial Nikkei index futures position you should take (whether a long or short position, and how many trading units). Assume that currently the delta of a put option with strike JPY 19,000 is −0.25.
[Note: If you didn't answer question d), assume a position of 2,010 put options.] (4 points)

g) You decide that a floor created by a put option with a strike of JPY 19,000 is too high and decide instead to create a floor using a put option with a strike of JPY 18,000. However, there are almost no put options with a strike of JPY 18,000 traded in the market, so you must use futures to synthesize a dynamic hedge.

How will you need to change your initial futures position at the current point in time to convert from a dynamic hedge futures position that synthesizes a floor created with a put option that has a strike of JPY 19,000 to a dynamic hedge that synthesizes a floor created with a put option that has a strike of JPY 18,000? Describe the trading directions to be taken with reasons. No calculations required. (4 points)

Question 5: Derivative Valuation and Analysis **(15 points)**

On November 1, 2018 the financial manager at Airline A of the United States was planning to purchase 1 million gallons of jet fuel at the end of 1 month from now. Concerned about a potential rise in the price of jet fuel, he decided to hedge with NYMEX heating oil futures, the standard hedge instrument for jet fuel. On November 1, 2018 the spot price of jet fuel was 2.30 dollars per gallon and the 1-month futures price was 2.40 dollars. Changes in the jet fuel spot price and NYMEX heating oil futures price from January to October 2018 are shown below.

Table 1: Changes in the jet fuel spot price and NYMEX heating oil futures price (Unit: dollars per gallon)

	Change in the heating oil futures price (ΔF_t)	Change in the jet fuel spot price (ΔS_t)
January 2018	0.034	0.024
February 2018	0.045	0.04
March 2018	0.046	0.03
April 2018	−0.047	−0.042
May 2018	0.002	0.005
June 2018	−0.039	−0.022
July 2018	−0.025	−0.01
August 2018	−0.029	−0.007
September 2018	0.02	0.027
October 2018	−0.003	0.007

Mean change in the futures price:	USD 0.00040
Standard deviation of change in the futures price:	USD 0.035
Mean change in the spot price:	USD 0.0052
Standard deviation of change in the spot price:	USD 0.026
Correlation coefficients between change in the futures price and change in the spot price:	0.96

a) Using S_t to represent the spot price of the hedged asset at time t and F_t to represent the futures price of the hedge instrument at time t, define basis and basis increase. Use the 10-month change in spot prices and change in futures prices above to explain whether the basis has increased or not. (5 points)

b) Calculate the optimal number of futures contracts and whether they should be sold or bought. Assume 42,000 gallons per futures contract and round the 1st decimal place to arrive at a whole number for your answer. As you do this, make use of the minimum variance hedge ratio in your calculation and round the hedge ratio to the 3rd decimal place. (5 points)

c) The example above does not provide a perfect hedge. Present two reasons why the example above does not provide a perfect hedge. (5 points)

Question 6: Portfolio Management **(28 points)**

You have been newly appointed as the executive director in charge of the fund management for the ABC Pension Fund. The fund currently engages in passive, diversified investments in global equities and global bonds.

Over the past 5 years when the previous director was in charge, central banks around the world maintained low-interest rate policies in order to tackle deflation and to stimulate economic recovery. Now, a new government has come to power in the United States, and the stock market has begun to rise on expectations of future tax cuts and economic stimulus.

Table 1 shows the market environment over the past 5 years, while Table 2 shows the market forecast for the first 5 years of your tenure, anticipating that central banks will bring an end to low-interest policies and interest rates will rise, but stock markets will continue to be bolstered by the economic recovery.

Table 1: Market environment for the past 5 years (actuals)

	Global equities	Global bonds
Return (p.a.)	13.0%	4.0%
Standard deviation of return (p.a.)	16.0%	7.0%
Correlation coefficients between returns	0.7	

Table 2: Market forecast for the next 5 years (forecasts)

	Global equities	Global bonds
Return (p.a.)	8.0%	−1.0%
Standard deviation of return (p.a.)	16.0%	7.0%
Correlation coefficients between returns	−0.3	

a) The current allocation between global equities and global bonds is 40% and 60%. Assuming constant weights, calculate the return and risk (standard deviation of return) for the past 5 years, and the expected return and risk for the next 5 years for the ABC Pension Fund portfolio as a whole. (6 points)

b) Answer the following questions regarding efficient frontier.

b1) By using expected returns and risks over the next 5 years in Table 2, explain and draw the efficient frontier in the graph assuming a correlation coefficient of +1. (2 points)

b2) If the correlation coefficient is −0.3 as you expect for the next 5 years, the asset allocation of the minimum variance portfolio will be 22% equities and 78% bonds. Based on this result, calculate the risk and expected return of the minimum variance portfolio, draw the approximate shape of the efficient frontier in a graph, and explain the impact of the correlation coefficient on the shape of the efficient frontier. (4 points)

b3) Suppose that Portfolio A has a risk level of 11% under the assumption of perfect correlation (+1). Using Portfolio A as the base case, compare Portfolio B, which has the same risk level on the efficient frontier when the correlation coefficient is −0.3, and Portfolio C, which has the same expected return on the efficient frontier when the correlation coefficient is −0.3 with Portfolio A. Comment on the diversification effect. Plot the portfolios A, B, C on a chart, with all the relevant components (efficient frontier, global bonds, global equities, minimum variance portfolio, horizontal and vertical axes, etc.). (4 points)

c) The fund currently adopts only passive investment, but you are considering active investment for global equities, in order to achieve higher returns.

c1) There are two approaches for calculating returns: the time-weighted rate (TWR) of return and the money-weighted rate of return (MWR). Which rate of return is a better measure of the fund active manager's skill? Explain. (3 points)

c2) Measuring an active manager's skill requires more than just a simple comparison of returns; risks must also be considered in order to measure the manager's investment efficiency. There are two approaches to measuring risk-adjusted investment performances: the Sharpe ratio and the information ratio. Which of these measures provides a better measure of active managers' skill in the same asset class, the Sharpe ratio or the information ratio? Provide the definitions of the two ratios and explain your answer. (5 points)

c3) Calculate the Sharpe ratio and the information ratio of the active manager who has on an annual basis a 5% portfolio return with a standard deviation of 10% and a tracking error of 5.0%. Assume that the benchmark return is 2.5% p.a. and the risk-free rate is 0.1% p.a. (4 points)

Question 7: Portfolio Management **(45 points)**

You manage a defined benefit corporate pension fund. Table 1 contains the present values of pension asset and pension liability, and also expected returns, risks (all annualized), and correlation coefficients assumed by your pension plan. The expected return on the pension liability is the same as the assumed interest rate of the pension plan.

Table 1: Pension asset and pension liability

	Present values	Expected returns	Risks (standard deviations)	Correlation coefficients	
				Pension asset	Pension liability
Pension asset	EUR 800	3.3%	5.5%	1.0	0.7
Pension liability	EUR 1,000	3.0%	7.0%	0.7	1.0

a) Using the information in Table 1, answer the following questions.

a1) Calculate the current funding ratio of the pension plan. (2 points)

a2) Calculate the expected surplus return against the pension liability.

[Note: if you didn't answer question a1), assume a funding ratio of 77%.] (3 points)

a3) Calculate the pension plan's surplus risk (standard deviation).

[Note: if you didn't answer question a1), assume a funding ratio of 77%).] (3 points)

a4) Suppose we define the "Surplus at Risk" as follows: There is a 5% chance that the pension plan's surplus (expressed in EUR) falls below the "Surplus at Risk" in one year. The "Surplus at Risk" is basically the "Value at Risk" applied to the pension surplus. Using the expected surplus return and surplus risk, determine the amount of the "Surplus at Risk". Show your steps.

[Hint: Use the standard normal distribution table in the formulae booklet.] (4 points)

Your pension asset in Table 1 is allocated between domestic bonds and global equities. Table 2 contains your pension plan's assumption about expected returns, risks, and the correlation coefficients with the pension liability for both asset classes.

Table 2: Breakdown of pension assets

	Present values	Expected returns	Risks (standard deviations)	Correlations with the pension liability
Domestic bonds	EUR 600	2.4%	4.5%	0.9
Global equities	EUR 200	6.0%	16.0%	0.2

Your long-term investment objective of pension plan management is to maintain a positive and stable surplus.

b) Use the information in Table 1 and Table 2 to answer the following questions.

b1) Describe briefly one advantage and one disadvantage of increasing the weight of domestic bonds and decreasing the weight of global equities. (4 points)

b2) Calculate how you need to allocate your pension asset (EUR 800) among domestic bonds and global equities to ensure the expected surplus return to be non-negative. (4 points)

c) Suppose that the expected return and risk assumptions for domestic bonds in Table 2 are based on realized return and realized risk during the past 5 years. Over the past 5 years, domestic interest rates have continued to decline, and today they are at historically low levels of virtually zero. You suspect that the assumptions in Table 2 are no longer valid. Answer the following questions in light of this.

c1) You believe that it is extremely unlikely for the domestic interest rates to decline further. Discuss briefly how you would modify the expected return assumption for the domestic bonds in Table 2. No calculations are necessary. (4 points)

c2) Suppose you change the assumed expected return and assumed risk for the domestic bonds as you have described in c1). How will this change affect the optimal allocation between domestic bonds and global equities in pension assets? Discuss briefly (1) from the perspective of the expected return and risk of pension assets, and (2) from the perspective of the expected return and risk of the pension surplus. (11 points)

d) As a substitute for domestic bonds, you consider investing part of the asset in foreign bonds with fully-hedged currency exposures (i.e., no currency exposures). To answer the following questions, assume that the uncovered interest parity holds (i.e. currency hedging strategies do not affect expected returns on foreign bonds).

d1) Even when the uncovered interest parity holds, risk averse investors may find it beneficial to hedge the currency exposures of foreign bonds only partially rather than to hedge the exposures fully. Explain a circumstance (or circumstances) in which the investors can benefit from taking some (unhedged) currency exposures in investing in foreign bonds. (5 points)

d2) Suppose that foreign bonds, with or without currency hedging, have higher expected returns than domestic bonds. Suppose further that fully-hedged foreign bonds and domestic bonds have the same risk levels. However, it may not be optimal to replace all domestic bonds with foreign bonds. Explain a circumstance (or circumstances) in which the pension fund can benefit from keeping the domestic bonds in its portfolio. (5 points)

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Solutions

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Question 1: Fixed Income Valuation and Analysis **(28 points)**

a)

$$P_0 = \frac{CF_t}{(1+y)^{t}} = \frac{100}{(1+1.75\%)^{30}} = 59.42 \qquad ①$$

$$D_{Mod} = \frac{D}{1+y} = \frac{30}{1+1.75\%} = 29.48 \qquad ②$$

$$C = \frac{1}{P} \cdot \frac{1}{(1+y)^2} \cdot \sum_{i=1}^{N} \frac{t_i(t_i+1) \cdot CF_i}{(1+y)^{t_i}}$$

For a zero coupon bond:

$$C = \frac{1}{P} \cdot \frac{1}{(1+y)^2} \cdot \frac{T(T+1) \cdot CF_T}{(1+y)^{T}} = \frac{T(T+1)}{(1+y)^2} \qquad ③$$

$$= \frac{30 \cdot 31}{(1+1.75\,\%)^2} = 898.28$$

b)

The absolute price change using modified duration and convexity is given by:

$$\Delta P = P \cdot \left[-D_{Mod} \cdot \Delta Y + \frac{1}{2} C \cdot (\Delta Y)^2 \right]$$

$$= 59.42 \cdot \left[-29.48 \cdot 1.25\,\% + \frac{1}{2} \cdot 898.28 \cdot (1.25\,\%)^2 \right] = -17.73$$

[Alternative answer: $= 60 \cdot \left[-29.5 \cdot 1.25\,\% + \frac{1}{2} \cdot 900 \cdot (1.25\,\%)^2 \right] = -17.91$]

c)

The approximate new price using only duration is given by:

$$\Delta P \cong -D_{Mod} \cdot P \cdot \Delta Y = -29.48 \cdot 59.42 \cdot 1.25\,\% = -21.9$$

$$\therefore P^{new} \cong P + \Delta P = 59.42 - 21.9 = 37.52$$

[Alternative answer: $-D_{Mod} \cdot P \cdot dY = -29.50 \cdot 60 \cdot 1.25\% = -22.13$
Pnew = P + dP = 60 − 22.13 = 37.87]

The exact new price:

$$P^{new} = \frac{100}{(1+3\,\%)^{30}} = 41.2$$

If duration and convexity were used, the new price would be:

$P^{new} = 59.42 - 17.73 = 41.69$

[Alternative answer: $P^{new} = 60 - 18 = 42$]

The result of new price using modified duration and convexity is fairly reliable as higher differentials/moments have very small impact. Using modified duration and convexity is more precise than using only modified duration.

d)

Let w_1 be the weight of the 4% Corporate bond, w_2 the weight of the 2% Covered bond and w_3 the weight of the zero coupon Government bond.

Then we have:

$$w_3 = 1 - w_1 - w_2 \qquad (1)$$

$$w_1 \cdot 6.04 + w_2 \cdot 8.92 + (1 - w_1 - w_2) \cdot 29.48 = 12 \qquad (2)$$

$$w_1 \cdot 36 + w_2 \cdot 73 + (1 - w_1 - w_2) \cdot 898 = 200 \qquad (3)$$

Then

$$w_1 \cdot (6.04 - 29.48) + w_2 \cdot (8.92 - 29.48) = 12 - 29.48$$

$$w_1 \cdot (36 - 898) + w_2 \cdot (73 - 898) = 200 - 898$$

Then

$$w_1 \cdot (23.44) + w_2 \cdot (20.56) = 17.48 \qquad (2')$$

$$w_1 \cdot (862) + w_2 \cdot (825) = 698 \qquad (3')$$

Solving we get:

$$w_1 = \frac{17.48 - w_2 \cdot 20.56}{23.44}$$

substituting in (3′)

$$\left(\frac{17.48 - w_2 \cdot 20.56}{23.44}\right) \cdot (862) + w_2 \cdot (825) = 698$$

$$\Rightarrow \quad w_2 = 80.1\ \%$$

$$\Rightarrow \quad w_1 = \frac{17.48 - 80.1\ \% \cdot 20.56}{23.44} = 4.3\%$$

$$\Rightarrow \quad w_3 = 1 - 4.3\% - 80.1\ \% = 15.6\ \%$$

[Alternative answer:

$$w_3 = 1 - w_1 - w_2 \quad (1)$$

$$w_1 \cdot 6.04 + w_2 \cdot 8.92 + (1 - w_1 - w_2) \cdot 29.5 = 12 \quad (2)$$

$$w_1 \cdot 36 + w_2 \cdot 73 + (1 - w_1 - w_2) \cdot 900 = 200 \quad (3)$$

$$w_1 \cdot (23.46) + w_2 \cdot (20.58) = 17.5 \quad (2')$$

$$w_1 \cdot (864) + w_2 \cdot (827) = 700 \quad (3')$$

Solving we get:

$$w_1 = \frac{17.5 - w_2 \cdot 20.58}{23.46}$$

substituting in (3′)

$$\left(\frac{17.5 - w_2 \cdot 20.58}{23.46}\right) \cdot (864) + w_2 \cdot (827) = 700$$

$$\Rightarrow \quad w_2 = 80.4\ \%$$

$$\Rightarrow \quad w_1 = \frac{17.5 - 80.4\ \% \cdot 20.58}{23.46} = 4.1\%$$

$$\Rightarrow \quad w_3 = 1 - 4.1\ \% - 80.4\ \% = 15.5\ \%\ ]$$

e)

The practical problem is that with three bonds and thus three unknowns (w) as well as two constraints (sum of weights = 1 and the portfolio's modified duration of 12) there are infinite number of solutions – and hence no clearly defined answer.

Also, hedging pension assets/liabilities solely based on modified duration only works for small parallel interest rate movements (i.e. bad hedge results for larger and/or non-parallel movements) and with a flat term structure.

Question 2: Fixed Income Valuation and Analysis (16 points)

a)
The price is given by:

$$P_0 = \frac{2\% \cdot 110}{(1+1\,\%)} + \frac{2\% \cdot 110}{(1+1\,\%)^2} + \frac{102\% \cdot 110}{(1+1\,\%)^3} = 110 \cdot 1.0294 = 113.24$$

b)
The real yield (R^{Real}) at both, the time of purchase and sale was 1%, therefore the real holding period return (HPR) for the 1-year period is 1%. The nominal holding period return is found by multiplying the real holding period return by the CPI growth rate:

$$\begin{aligned} HPR &= (1+R^{Real}) \cdot (1+\text{inflation}) - 1 \\ &= (1+1.0\,\%) \cdot (1+1.5\,\%) - 1 \\ &= 1.01 \cdot 1.015 - 1 \cong 2.52\,\% \end{aligned}$$

[Alternative answer: R^{Real} + Inflation Rate = 1.00% + 1.50% = 2.50%, can also be accepted as percentage changes are small]

[Alternative answer:
The real price after 1 year is:

$$P_1^{Real} = \frac{2}{(1+1\,\%)} + \frac{102}{(1+1\,\%)^2} = 101.97$$

$$\text{CPI index ratio} = 1.10 \cdot 1.015 = 1.1165$$

$$P_1^{Nominal} = 101.97 \cdot 1.1165 = 113.85$$

The coupon earned is $2\,\% \cdot 111.65 = 2.233$

Therefore, Holding period return is

$$HPR = \frac{2.233 + 113.85}{113.24} - 1 \cong 2.52\,\%]$$

c)

$$1+R^{Nom} = (1+R^{Real}) \cdot (1+ \text{Expected Inflation Rate})$$

$$\text{Expected Inflation Rate} = \frac{1+R^{Nom}}{1+R^{Real}} - 1$$

$$= \frac{1+2\,\%}{1+1\,\%} - 1 = 1\,\%$$

d)

If the rate of inflation is greater than 0.99%, the nominal yield on the inflation-indexed bond will be higher than the return on the government bond, since:

Nominal yield of the standard government bond: 2.00%

With $R^{Real} = 1\%$,

We have: (1+Nominal yield of the linker) = $(1+R^{Real}) \cdot$ (1+Expected Inflation Rate) > (1+2%)

→ Expected Inflation Rate > 0.99%

[Alternative answer:
Nominal yield of the linker = R^{Real} + Expected Inflation Rate > 2%
→ Expected Inflation Rate > 2% − 1% = 1%.]

e)

First reason: Liquidity. Inflation-indexed government bonds generally lack liquidity. This, however, lowers their price and therefore increases their yield.

Second reason: Generally speaking, inflation rate risk will be lower for indexed bonds relative to conventional bonds due to the linking of interest and redemption values to inflation measure, although the linking occurs with a slight lag. However, there is no guarantee that the CPI growth rate will perform as expected, and because of this risk of fluctuation, both the prices of inflation-indexed and conventional government bonds will be correspondingly lower, and their yields higher than if the CPI were not subject to such fluctuations or differences between expectations and realizations.

Question 3: Fixed Income Valuation and Analysis **(15 points)**

a)
The conversion price is given by:

$$\text{Conversion price} = \frac{\text{Face value of the convertible bond}}{\text{Number of shares per bond (if there is a conversion)}}$$

$$= \frac{10{,}000}{44.5514} = \text{EUR } 224.4598$$

b)
First, conversion value is given by:

$$\text{Conversion value } (\text{Parity}) = \text{Conversion ratio} \cdot \text{Market price of stock}$$

$$= 44.5514 \cdot 162 = \text{EUR } 7{,}217.33$$

The conversion premium is given by:

$$\text{Conversion premium } (\text{in \%}) = \frac{\text{Market price of bond} - \text{Conversion value}}{\text{Conversion value}}$$

$$= \frac{10{,}216 - 7{,}217.33}{7{,}217.33} = 41.55\% \quad \rightarrow \quad > 20\%$$

The traditional premium analysis is a convertible investment strategy, which consists in selecting the security only if the premium is less than 20%. In the above case, we observe that the premium is considerably larger, which means that - according to the criteria - this convertible bond will not be selected.

c)
The straight value (or pure debt value) of the convertible bond can be computed by applying the yield to maturity of the similar straight bond to our convertible bond. As the two securities are rather similar (except for the coupon rate) we can neglect such difference. Then, the straight value of the convertible is determined by:

$$P_0 = \sum_{i=1}^{N} \frac{CF_i}{(1+R_i)^{t_i}}$$

$$= \frac{200}{(1+4.25\%)} + \frac{200}{(1+4.25\%)^2} + \frac{200}{(1+4.25\%)^3} + \frac{200}{(1+4.25\%)^4} + \frac{10200}{(1+4.25\%)^5}$$

$$= 9{,}005.34$$

d)
The payback period of the convertible bond is given by:

$$PP = \frac{\text{Conversion premium}}{(CY - DY)}$$

$$= \frac{41.55\ \%}{\left(\frac{2\ \%}{102.16\ \%} - 0.9\%\right)} = 39.28 \text{ years}$$

where
CY: current yield of the convertible
DY: dividend yield on the common stock

[Alternative answer: with a conversion premium of 40%:

$$PP = \frac{40\ \%}{\left(\frac{2\ \%}{102.16\ \%} - 0.9\%\right)} = 37.82 \text{ years}\ ]$$

The payback period indicates how quickly an investor is able to recover the conversion premium paid for the convertible through the higher current income. A short payback period is considered a good bet for investment. In the above case, about 40 years is a rather long payback period. Investors would therefore typically not invest in the security, at least from a payback analysis point of view.

e)
The convertible bond, which is generally called "exchangeable bond", is rated AA as MunichRe Finance, a well-established insurance group, is the issuer. SAP, in contrast, is an industrial company with more business risk. In fact, current bonds issued by SAP are rated A. If SAP had issued the convertible directly, the rating would have likely been lower and the YTM higher.

Question 4: Derivative Valuation and Analysis and Derivatives in PM (33 points)

a)

If dividends and transaction costs can be ignored, the no-arbitrage condition produces the following relation between the futures price F_0 and the spot price S_0 with a risk-free rate per annum r_f and time to maturity τ years.

$F_0 = S_0 \cdot (1+r_f \cdot \tau) = 20{,}000 \cdot (1+4\,\% \cdot 0.25) = \text{JPY } 20{,}200.$

b)

The put-call parity says: $C_0(K) - P_0(K) = \frac{F_0 - K}{1+r_f \cdot \tau}$

or $C_0(K) - P_0(K) = S - \frac{K}{1+r_f \cdot \tau}$ with $F_0 = S \cdot (1 + r_f \cdot \tau) \leftrightarrow S = F_0 / (1 + r_f \cdot \tau)$

$C_0(K) =$ JPY 897; $P_0(K) =$ JPY 678; $F_0 =$ 20,200; $r_f =$ 4%; and $\tau =$ 0.25 years.

Applying these values to the put-call parity, the left side becomes $C_0(K) - P_0(K) = 897 - 678 = \text{JPY } 219$ and the right side $\frac{F_0 - K}{1+r_f \cdot \tau} = \frac{200}{1.01} = \text{JPY } 198.$

The put-call parity does not hold. Therefore, there is an arbitrage opportunity.

[Alternative answer:
We could also calculate the theoretical call price: JPY 876, showing that the listed call is overpriced at JPY 897. Or, theoretical put price: JPY 699, showing that the listed put is underpriced at JPY 678.]

To institute an arbitrage, you sell 1 trading unit of call options, purchase 1 trading unit of put options, take a 1 trading unit position in long futures, and lend the present value of the difference between the futures price and the strike price at the risk-free rate. The payoff from this trade is zero at maturity, but now you can obtain the arbitrage profit of

$$1000 \cdot \left(C_0(K) - P_0(K) - \frac{F_0 - K}{1+r_f \cdot \tau} \right) = 1000 \cdot (219 - 198) = \text{JPY } 21{,}000$$

[Note: If calculated at expiry, the arbitrage profit becomes: 21,000 · (1 + 4% · 0.25) = JPY 21,210.]

[Alternative answer:
As Call is overpriced in the market: sell the Call, and buy a synthetic Call (Buy Put, Buy underlying asset, lend/borrow the money needed – As underlying asset is not listed, Buy the future instead)

Sell Call	+897
Buy Put	−678
Buy Futures	−20,200 (futures → no cash)
Cash lent (897−678)	−219
Net cash at beginning	0.00

At expiry:

	S<20,000	S>20,000
Short Call expiry	0	− (S − 20,000)
Long Put expiry	20,000 − S	0
Long Future expiry	S − 20,200	S – 20,200
Lend expiry (4% 3M)	+221.19	+221.19
Profit	+21.19	+21.19

With nominal = 1,000 → arbitrage profit = JPY 21,190]

[Alternative solution with future price JPY 20,100
As Call is overpriced in the market: sell the Call, and buy a synthetic Call (Buy Put, Buy underlying asset, lend/borrow the money needed – As underlying asset is not listed, Buy the future instead)

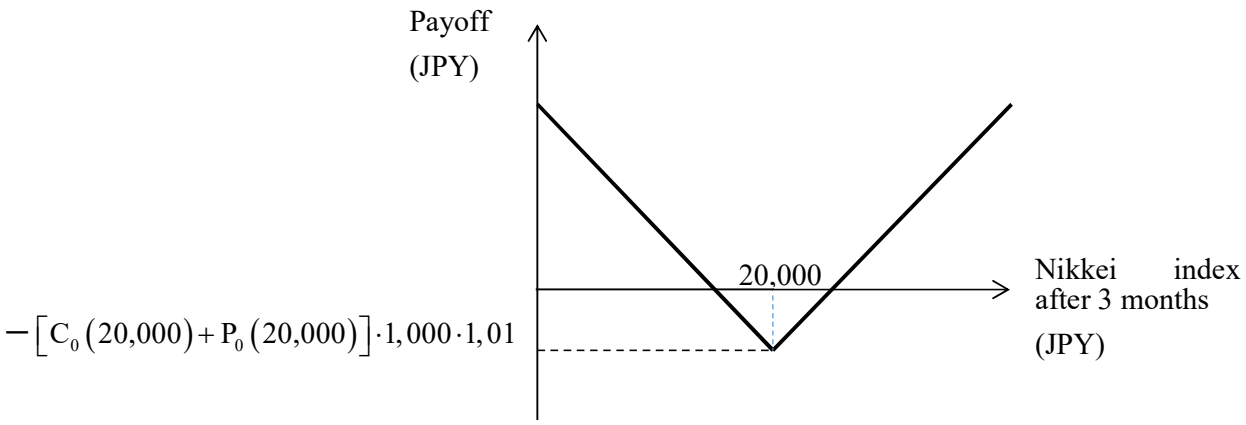
Sell Call	+897
Buy Put	−678
Buy Futures	−20,100 (futures → no cash)
Cash lent (897−678)	−219
Net cash at beginning	0.00

At expiry:

	S<20,000	S>20,000
Short Call expiry	0	− (S − 20,000)
Long Put expiry	20,000 − S	0
Long Future expiry	S − 20,100	S – 20,100
Lend expiry (4% 3M)	+221.19	+221.19
Profit	+121.19	+121.19

With nominal = 1,000 → arbitrage profit = JPY 121,190]

c)
One example is a long straddle that purchases 1 trading unit each of the put option and the call option with strike price K= JPY 20,000. Denoting the prices of such call and put options as $C_0(20{,}000)$ and $P_0(20{,}000)$, respectively, the long straddle position costs JPY $\left[C_0(20{,}000)+P_0(20{,}000)\right]\cdot 1{,}000$. The amount borrowed at the risk-free rate and repaid at maturity is the maximum loss amount at the maturity of the long straddle. The payoff diagram for this position at maturity is as shown below. Therefore, the position will generate a profit if there is a large move in the Nikkei index either upward or downward.

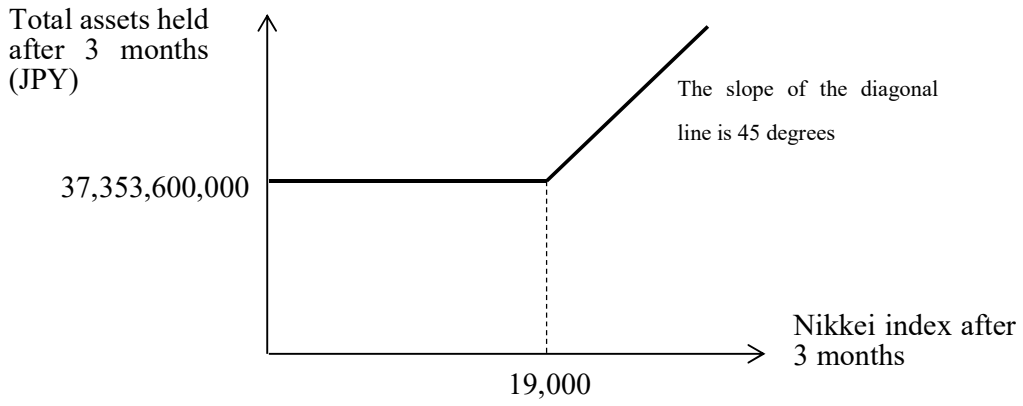


d)
If, at the end of 3 months, the Nikkei index is below the strike of JPY 19,000, achieving a certain level of total assets held from in-the-money put options requires a position in which the return from the put options offsets the loss on the stock portfolio. The value of the stock portfolio is JPY 40 billion, which is 2 million times the current Nikkei index of JPY 20,000. One trading unit of the put options is 1,000 times the Nikkei index, so the required put option position is 2,000 (= 2 million / 1,000) trading units of a long position.

e)
The floor for total assets held achieved after 3 months is calculated by subtracting the repayment of the put option purchase price after 3 months from the floor achieved through a combination of the stock portfolio and put options.
Hence: $19{,}000\cdot 1{,}000\cdot 2{,}000 - 320\cdot 1{,}000\cdot 2{,}000\cdot(1+4\ \%\cdot 0.25) = \text{JPY}37{,}353{,}600{,}000$

[Alternative Method:
Put creates a floor at 19,000 → −5% on the index (today's price 20,000)
−5% on 40bn portfolio → 38,000 M value
Price paid for 2,000 puts (quantity 1,000) @ 320JPY, with 4% interests at 3 months:
$2{,}000\ \cdot\ 1{,}000\ \cdot\ 320\ \cdot\ (1+\ 4\ \%\ \cdot\ 0.25)\ =\ 646.4M$
$38{,}000\ M\ -\ 646.4\ M\ =\ \text{JPY}\ 37{,}353{,}600{,}000\ ]$

The following graph shows the relationship between total value of assets held and the Nikkei index after 3 months.



f)

The futures delta is $\frac{\partial F_0}{\partial S_0} = 1 + 4\% \cdot 0.25 = 1.01$, and the delta of the put option with a strike price of JPY 19,000 is $\frac{\partial P_0(19{,}000)}{\partial S_0} = -0.25$. For a dynamic hedge, it is necessary that the delta of the put option position match the delta of the futures position, which requires a futures position of $-\frac{0.25}{1.01} = -0.248$ trading units per unit put option (a short (selling) position of 0.248).

[Alternative: −0.25 / 1.01 = 0.2475 or 0.248 depending on rounding]

To create the floor in e), you purchase 2,000 trading units of put options, which means that the futures position for the delta hedge is $2{,}000 \cdot (-0.248) = -495$ trading units (or −496). Therefore, the initial futures position to be taken to achieve the dynamic hedge is a 495-unit short (selling) position.

[Alternative: 2,000 · −0.2475 = −495 /// 2,000 · −0.247 = −494 depending on rounding]

g)

If the time to maturity and the price of the underlying asset remain unchanged, the lower the strike price is, the smaller the absolute value of the put option delta, because the put option price is less sensitive to the underlying asset price. (The put option delta is negative, resulting in a larger value, like −0.10 to −0.15). For a dynamic hedge, the delta of the options position must match the delta of the futures position. Therefore, the size of the futures short (selling) position in a dynamic hedge that can synthesize a floor using put options with a strike of JPY 18,000 is smaller than the futures short (selling) position of a dynamic hedge to synthesize a floor using put options with a strike of JPY 19,000. The switch will therefore require a reduction of the short futures position, by buying back some futures contracts.

Question 5: Derivative Valuation and Analysis **(15 points)**

a)
Basis is defined as $S_t - F_t$. The definition of basis increase is $S_2 - F_2 > S_1 - F_1$
$\Leftrightarrow S_2 - S_1 > F_2 - F_1$. Since the average change in spot prices has been greater than the average change in futures prices [0.0052>0.0004] the basis has increased.
The responses $F_t - S_t$, rate of change $(S_t - F_t)/S_t$ and $(F_t - S_t)/St$ are all correct.
[Note to the corrector: Other answers are also correct if they are consistent with the definition of basis.]

b)
The minimum variance hedge ratio (HR) is the value found by multiplying the correlation coefficients by the standard deviation of the change in underlying asset prices divided by the standard deviation of the change in futures prices.

$$\text{HR} = \frac{\text{Cov}(\Delta S, \Delta F)}{\text{Var}(\Delta F)} = \rho_{\Delta S, \Delta F} \cdot \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}}$$

$$= 0.96 \cdot \frac{0.026}{0.035} = 0.71$$

The optimal number futures is found by multiplying the minimum variance hedge ratio by the size of the asset to be hedged and then dividing by the size of the futures contract that is the hedge instrument.

$$N_F = \text{HR} \cdot \frac{N_s}{k}$$

$= 0.71 \cdot 1$ million / 42,000 ≈ 17

In this case, the futures should hedge the price movement of jet fuel to be bought, therefore, 17 futures should be bought.

c)
- Because the asset being hedged does not match exactly the asset underlying the futures contract served as the hedge instrument.

- Because the time at which the hedged asset is bought or sold may not necessarily correspond to the delivery month of the futures contract that serves as the hedge instrument.

- Because the number of futures contracts is rounded: there is a size mismatch.

- Because commodities futures have unforeseeable convenience yields.
- This is based on limited historical observations, which may result in estimation errors.

Question 6: Portfolio Management **(28 points)**

a)

Return of overall portfolio: $\mu_P = w_E \cdot \mu_E + w_B \cdot \mu_B$

Risk of overall portfolio: $\sigma_P = \sqrt{w_E^2 \cdot \sigma_E^2 + 2 \cdot w_E \cdot w_B \cdot \sigma_E \cdot \sigma_B \cdot \rho_{E,B} + w_B^2 \cdot \sigma_B^2}$

Therefore:

- Past 5 years

Return: $\mu_A = 0.4 \cdot 13\,\% + 0.6 \cdot 4\,\% = 7.6\,\%$

Risk: $\sigma_A = \sqrt{(0.4 \cdot 16\,\%)^2 + 2 \cdot 0.4 \cdot 0.6 \cdot 16\,\% \cdot 7\,\% \cdot 0.7 + (0.6 \cdot 7\,\%)^2} = 9.81\,\%$

- Next 5 years

Expected return: $\mu_A = 0.4 \cdot 8\,\% + 0.6 \cdot (-1\,\%) = 2.6\,\%$

Risk: $\sigma_A = \sqrt{(0.4 \cdot 16\,\%)^2 + 2 \cdot 0.4 \cdot 0.6 \cdot 16\,\% \cdot 7\,\% \cdot (-0.3) + (0.6 \cdot 7\,\%)^2} = 6.52\,\%$

b)

b1)

If the correlation coefficient is +1, there is no diversification effect and the efficient frontier is a straight line that joins the risk-return for global equities and global bonds.

[See chart in solutions b3); candidate should draw the chart with points Global Equities (8;16) and Global Bonds (−1;7), and the straight segment between these two points. Axes are Expected Return (E(R)) and risk (σ).]

b2)

Return of the minimum variance portfolio: $\mu_{MinVar} = 0.22 \cdot 8\,\% + 0.78 \cdot (-1\,\%) = 1.0\,\%$

[with two decimals, return = 0.98%]

Risk of the minimum variance portfolio:

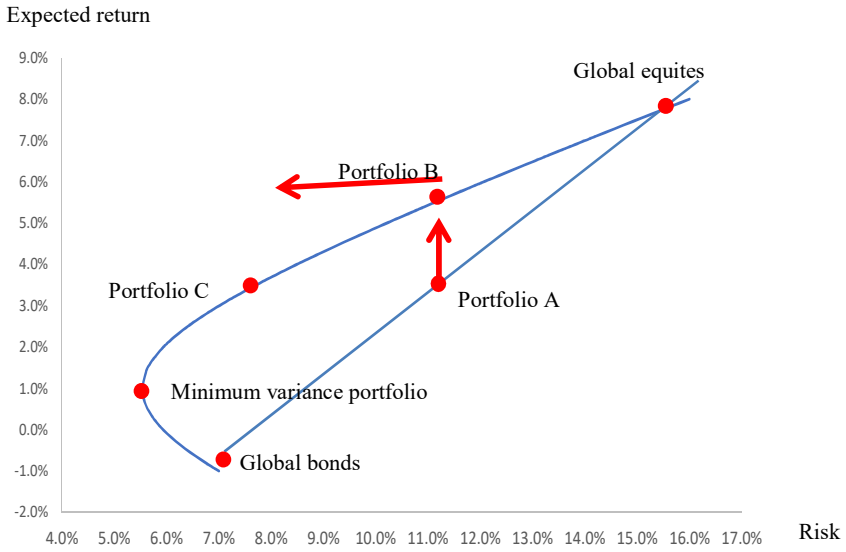
$\sigma_{MinVar} = \sqrt{(0.22 \cdot 16\,\%)^2 + 2 \cdot 0.22 \cdot 0.78 \cdot 16\,\% \cdot 7\,\% \cdot (-0.3) + (0.78 \cdot 7\,\%)^2} = 5.54\,\%$

As the correlation coefficient is negative, the diversification will be effective and the efficient frontier will be a convex curve in the upper left.

[See chart in solution b3); candidate should draw the convex curve, and the point for Minimally Diversified Portfolio (Min Variance Ptf) at 1.0% return / 5.54% risk.]

b3)
As a result, Portfolio B, which has the same risk level as Portfolio A, will have higher returns than Portfolio A. Portfolio C, which has the same return level as Portfolio A, will have lower risk than Portfolio A. Hence, the negative correlation enhances Portfolio A characteristics.

Graph: Efficient Frontier



c)
c1)
It is better to use the time-weighted rate of return for measuring an active manager's skill.

Rationale:
The money-weighted rate of return is influenced by cash flow, which the manager cannot control. So, the time-weighted rate of return, which is not influenced by cash flow, should be used.

c2)
The Sharpe ratio provides a measure of absolute risk and is useful in determining which among different types of investment will have the higher returns for the same risk levels.

$$\text{Sharpe ratio} = \frac{\text{portfolio return} - \text{risk-free rate}}{\text{portfolio risk}}$$

The information ratio is calculated by dividing excess return (active return) by tracking error and is useful in comparing managers with the same risk levels or same asset classes (benchmark).

$$\text{Information ratio} = \frac{\text{portfolio return} - \text{benchmark return}}{\text{tracking error}}$$

So, the information ratio is more adequate to measure active managers' skills.

c3)

$$\text{Sharpe ratio} = \frac{5\,\% - 0.1\,\%}{10\,\%} = 0.49$$

$$\text{Information ratio} = \frac{5\,\% - 2.5\,\%}{5\,\%} = 0.5$$

Question 7: Portfolio Management **(45 points)**

a)

a1)

The current funding ratio FR_0 is calculated from the amount of pension asset A_0 and amount of pension liability L_0, as shown below.

$$FR_0 = \frac{A_0}{L_0} = \frac{800}{1,000} = 80\ \%$$

a2)

Expected surplus return against pension liability μ_S is calculated from the funding ratio FR_0, and expected return of pension asset and pension liability (μ_A, μ_L), as shown below.

$$\begin{aligned}\mu_S &= FR_0 \cdot \mu_A - \mu_L \\ &= 80\ \% \cdot 3.3\ \% - 3\ \% \\ &= -0.36\ \%\end{aligned}$$

[Alternative answer if a1) not answered:
using the given FR_0 = 77%; 77% · 3.3 − 3% = −0.46%]

a3)

The pension plan's surplus risk (standard deviation) σ_S is calculated from the funding ratio FR_0, risk of pension asset and pension liability (σ_A, σ_L), and correlation coefficient of pension asset and pension liability ρ_{AL}, as shown below.

$$\begin{aligned}\sigma_S &= \sqrt{(FR_0 \cdot \sigma_A)^2 - 2 \cdot \rho_{AL} \cdot (FR_0 \cdot \sigma_A) \cdot \sigma_L + \sigma_L^2} \\ &= \sqrt{(80\ \% \cdot 5.5\ \%)^2 - 2 \cdot 0.7 \cdot (80\ \% \cdot 5.5\ \%) \cdot 7\ \% + (7\ \%)^2} \\ &= 5.0239\ \%\end{aligned}$$

[Alternative answer if a1) not answered:
using the given FR_0 = 77%; solution = 5.0430%]

a4)
In standard normal distribution, the probability of having a value smaller than 1.65 standard deviations is 95%, so assuming that returns follow a normal distribution, the amount of value at risk (surplus at risk) for a period of 1 year $\mathrm{VaR}_S(95\,\%)$ is calculated from the amount of liabilities L_0, expected returns of the surplus μ_S, and surplus risk (standard deviation) σ_S, as shown below.

$$\begin{aligned}\mathrm{VaR}_S(95\,\%) &= L_0 \cdot (\mu_S - 1.65 \cdot \sigma_S) \\ &= 1000 \cdot (-0.36\,\% - 1.65 \cdot 5\,\%) \\ &= -86.1\end{aligned}$$

[Alternative using the answer calculated in a2):
with the given FR_0=77%; VaR(95%) = 1000. (−0.46% − 1.65.5%) = −87.1]

b)
b1)
Advantage: Domestic bonds have a correlation coefficient of 0.9 against pension liability, which is high, so surplus risk can be expected to decline if the weight of domestic bonds is increased.
Disadvantage: The plan currently has a funding ratio of 80%, and is therefore in shortfall, but the expected return on domestic bonds is lower than the expected return on pension liability, so increasing the weight of domestic bonds would reduce the likelihood of returning the pension surplus to positive territory and therefore contradicts the goals of pension asset management.

b2)
The amount of pension liability is EUR 1,000 and their expected returns are 3.0%, so the expected increase for pension liability each year is as shown below.

$$1,000 \cdot 3\,\% = 30$$

Because there is only EUR 800 in pension asset, representing the amount invested in domestic bonds as X and the amount invested in global equities as $(800 - X)$, the following must be satisfied in order to be able to expect return on par with the expected increase in pension liability.

$$X \cdot 2.4\,\% + (800 - X) \cdot 6\,\% = 30$$

$$\therefore X = \frac{800 \cdot 6\,\% - 30}{6\,\% - 2.4\,\%} = 500$$

In other words, the pension fund has to invest EUR 500 in domestic bonds and $(800 - 500) = \text{EUR } 300$ in global equities.

c)

c1)

Domestic interest rates have continued to decline for the past 5 years, so current interest-rate levels and bond yields can be assumed to be lower than the average values for the last 5 years. If there is little likelihood of further declines in interest rates, then it is unlikely to get as much capital gains from bonds as in the last 5 years. Because of this, it would be reasonable to reduce the expected return for domestic bonds below what is assumed in Table 2, for example, at around the same level as current bond yields.

c2)

(1) From the perspective of the expected return and risk of pension asset, the expected return of domestic bonds will be lower than initially assumed and the downside risk for domestic bond returns (interest rate rise risk) will be higher. This reduces the optimum investment weight for domestic bonds and increases the optimum investment weight for global equities by the same amount.

(2) From the perspective of the expected return and risk of the pension surplus, there is a greater possibility that pension liability, which has a high correlation with domestic bonds, will decrease (sharp increase in interest rates) rather than increase (sharp decline in interest rates). This means that the risk of a significant worsening of the surplus due to fluctuations in pension liability will be lower than initially assumed, and there will be relatively less need to hedge against pension liability with domestic bonds, which has a risk-return profile worse than initially assumed. The optimum investment weight of domestic bonds therefore declines, and the optimum investment weight of global equities increases by the same amount.

d)

d1)

Foreign-exchange returns are not perfectly correlated with domestic bonds, global equities, or other assets, and the diversification effect may reduce the risk in the portfolio. For example, if there is a negative correlation between domestic bonds and foreign exchange returns, partially removing the hedge on the foreign exchange exposure of foreign bonds may reduce the risk in the portfolio as a whole, and in this situation, it would be reasonable for a risk-averse investor to decide not to fully hedge foreign exchange exposures.

d2)

Even if the expected returns for foreign bonds for which foreign exchange rates have been fully hedged are higher than the expected returns from domestic bonds while risk levels are the same, the correlation coefficient against liability is not as high for fully hedged foreign bonds as it is for domestic bonds, and therefore, fully hedged foreign bonds will have higher surplus risk. The pension plan's goal is to keep surplus risk low over the medium term to long term, and therefore it could be rational not to replace all domestic bonds with fully hedged foreign bonds.