Pilot Questions - 2021

Paper 3.0(A)
Section B (Theory)
Set 1

Question 1 – Corporate Finance

1a) Discuss whether or not an increase in dividends is likely to benefit the shareholders of a publicly quoted company. **(6 marks)**

b) Beko Ltd is a highly geared company that wishes to expand its operations. Six possible capital investments have been identified, but the company only has access to a total of ₦620,000. The project are not divisible and may not be postponed until a future period. After the projects end it is unlikely that similar investment opportunities will occur.

Expected net cash inflows (including salvage value)

Project	Year 1 ₦	2 ₦	3 ₦	4 ₦	5 ₦	Initial Outlay ₦
A	70,000	70,000	70,000	70,000	70,000	246,000
B	75,000	87,000	64,000			180,000
C	48,000	48,000	63,000	73,000		175,000
D	62,000	62,000	62,000	62,000		180,000
E	40,000	50,000	60,000	70,000	40,000	180,000
F	35,000	82,000	82,000			150,000

Project A and E are mutually exclusive. All projects are believed to be of similar risk to the company's existing capital investments. Any surplus fund may be invested in the money market to earn a return of 9% per year.
The money market may be assumed to be an efficient market.
Beko's cost of capital is 12% per year.

Required:

b) Calculate:
  i. the expected net present value; **(8 marks)**
  ii. the expected profitability index associated with each of the six projects, and rank the projects according to both of these investment appraisal methods. Explain briefly why these rankings differ. **(4 marks)**

c) Give reasoned advice to Beko Ltd recommending which projects should be selected. **(4 marks)**

(Total: 22 marks)

Question 2 – Equity Valuation & Analysis

2. You are given the following facts about a company.

- Current stock price = ₦56.94
- Estimated earnings per share for the current year = ₦1.837
- Dividend for current year = ₦0.575
- Dividend growth rate = 8.18%
- Risk-free rate = 5.34%
- Equity risk premium = 5.32%
- Beta = 0.83

Required:

a) What are the justified trailing and leading P/E$_S$ based on the Gordon growth model?

b) Based on the justified trailing P/E and the actual P/E, is the company fairly valued, overvalued or undervalued? **(8 marks)**

Question 3 – Equity Valuation & Analysis

3. Emilia Kay, an equity analyst at Kayode Research Group (KRG), begins valuing Top Hotels plc (THP), a thinly and infrequently traded stock currently selling at ₦180 per share.

For estimating THP's required return on equity, Emilia uses the capital asset pricing model (CAPM) approach, but she thinks its own equity beta of 1.20 is not very reliable because of the stock's extremely thing trading volume. Therefore, she obtains the beta and other pertinent data for Excellent Hotels plc (EHP) (see Table 1), a midsized company in the same industry with high market liquidity trading on the Nigeria Stock Exchange.

Table 1: Comparative data for valuation

	THP	EHP
Equity beta	1.20	1.60
Debt ratio (Debt/Total	0.20	0.60

Emilia has also obtained the following forecasts on THP for possible investment decision **(see Table 2)**.

Table 2: Key investment statistics on THP

Return on equity (ROE)	12%
Retention rate	55%
Book value per share (BVPS) at the end of last year	₦200
Risk-free rate	4.40%
Stock market risk premium	9.00%
THP's current market price	₦180

a) Using relevant data from the above tables, calculate the **required** return on THP's equity, using CAPM. **(6 marks)**

b) Using the constant dividend growth-model, calculate the **expected** rate of return (i.e. the implied return) on THP's shares at the current share price. Then, discuss whether THP's shares are trading at a discount or at a premium if the CAPM holds.

(Note: For those who could not answer question (a), assume required return on equity of 11.6%) **(6 marks)**

c) The investment manager at KRG uses a different valuation approach. He calculates the theoretical share price by adding the present value (PV) of the future residual income per share to current book value per share using the following formulae:

Theoretical share price = (BVPS at end of last year) + (PV of residual income per share in and after this year)

Residual income per share =

EPS - (BVPS at the beginning of the year) × (Required rate of return on equity)

You are required to show the calculations following the investment manager's approach.

Then discuss whether THP 's shares are trading at a discount or at a premium.

(6 marks)

(Total: 18 marks)

Question 4 – Fixed Income Valuation & Analysis

4. A 30-year maturity bond has a 7% coupon rate, paid annually. It sells today for ₦867.42. A 20-year maturity bonds has 6.5% coupon rate, also paid annually. It sells today for ₦879.50. A bond market analyst forecasts that in 5 years, 25-year maturity bonds will sell at yields to maturity of 8% and 15-year maturity bonds will sell at yields of 7.5%. Because the yield curve is upward sloping, the analyst believes that coupons will be reinvested in short-term securities of 6%.
Which bonds offers the higher expected annualised rate of return over the 5-year period? **(6 marks)**

Question 5 – Fixed Income Valuation & Analysis

5. You have the following cash flow structure for the next 10 years:

   i. You have to pay back a loan of ₦1,000,000 at the end of 10 years.

   ii. You have to receive payments of ₦150,000 each from individual zero-coupon bond investments at an interval of every 2 years beginning end of 2nd year for the next 10 years (for a total of 5 payments).

   The market yield is assumed to be 5% on all the instruments and for all the maturities.

   For calculations use precision up to 2 decimal places.

   Required

   a) For your assets and liabilities as mentioned above, calculate the present value of the cash flows. What is the net surplus you have today? **(4 marks)**

   b) Calculate the Macaulay duration for your asset and the liability sides. **(3 marks)**

   c) Will there be any change in the net value of your assets and liabilities if there is an upward shift in the term structure of the interest rate by 100 basis points? Approximately calculate the surplus value and interpret the result. **(4 marks)**

   d) Consider you want to immunize your net position, as calculated in a) above, against the interest rate shifts by changing the pay-back period. Calculate the new duration for repayment of the loan assuming that the current market value of your liability remains unchanged. **(4 marks)**

   e) Will the immunization as done above remain appropriate forever without rebalancing? Justify your answer. **(3 marks)**

   (Total: 18 marks)

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Set 1

Solution 1 – Corporate Finance

1a) Differing views exist about the effect of dividends on a company's share price. Several authors, including Modigliani and Miller (M & M) have argued that dividend policy is irrelevant to the value of a company. Such arguments are formulated under very restrictive assumptions. If such conditions existed then shareholders would not value an increase in dividend payments. However, there are several real world factors that are likely to influence the preference of shareholders towards dividends or retentions (and hence expected capital gains). These include:

i) Taxation. In some countries dividends and capital gains are subject to different marginal rates of taxation, usually with capital gains being subject to a lower level of taxation than dividends.

ii) Brokerage fees. If shareholders have a preference for some current income and are paid no or low dividends their wealth will be reduced if they have to sell some of their shares and incur brokerage fees in order to create current income. Shareholders, especially institutional shareholders, often rely on dividends to meet their cash flow needs.

iii) The corporate tax treatment of dividends may favour a higher level of retention.

iv) If the company needs to finance new investment it is usually cheaper to use retained earnings. This is because most forms of external finance involve issue costs, which, in the case of equity finance can be three percent or more of the funds raised.

v) Information asymmetry may exist between shareholders and directors. If the market is not strong form efficient shareholders may have less complete knowledge of the likely future prospects of the company than directors, which may influence the shareholders' desire for dividends or capital gains.

The implications of an increase in dividends need to be considered by the company. Dividends are often regarded as an unbiased signal of a company's future prospects, an increase in dividends signalling higher expected earnings. A company should be careful to inform its shareholders of the reason for any increase in dividends.

A further factor is the use that the company can make of funds. If the company has a number of possible positive NPV investments then shareholders will normally favour undertaking these investments (at least on financial grounds), as they will lead to an increase in shareholder wealth. As previously mentioned, internal finance is cheaper than external finance and, ceteris paribus, would lead to a preference for

retentions. If, however, the company has relatively few projects and can only invest surplus cash in money market or other financial investments at an expected zero NPV, relative high dividends or share repurchase might be preferred.

b) i) and ii) Calculations of expected net present value and profitability indices

Project A

NPV = ₦70,000 × 3.605 − ₦246,000 = ₦6,350

$$\text{Profitability index} = \frac{\text{present value of cash inflows}}{\text{present value of cash outflows}} = \frac{252,350}{246,000} = 1.026$$

Project B

NPV = ₦75,000 × 0.895 + ₦87,000 × 0.797 + ₦64,000 × 0.712 − ₦180,000

= ₦1,882

$$\text{Profitability index} = \frac{181,882}{180,000} = 1.010$$

Project C

NPV = ₦48,000 × 1.69 + ₦63,000 × 0.712 + ₦73,000 × 0.66 - ₦175,000

= (₦2,596)

$$\text{Profitability index} = \frac{172,404}{175,000} = 0.985$$

Project D

NPV ₦62,000 × 3.037 − ₦180,000 = ₦8,294

$$\text{Profitability index} = \frac{188,294}{180,000} = 1.046$$

Project E

NPV ₦40,000 × 0.893 + ₦50,000 × 0.797 + ₦60,000 × 0.712 + ₦70,000 × 0.636 + ₦40,000 × 0.567 − ₦180,000 = ₦5,490

$$\text{Profitability index} = \frac{185,490}{180,000} = 1.031$$

Project F

NPV = ₦35,000 × 0.893 + ₦82,000 × 1.509 - ₦150,000 = ₦4,993

$$\text{Profitability index} = \frac{154,993}{150,000} = 1.033$$

Project Ranking	NPV	PI
1	D	D
2	A	F
3	E	E
4	F	A
5	B	B
6	C	C

The profitability index shows the present value per N of incremental outlay, and is a relative measure. NPV is an absolute measure showing the

expected benefit from a project. If projects differ in the amount of capital outlay, as they do in this case, NPV and PI may give different rankings.

iii) The projects selected should be the combination of projects with the greatest total NPV, subject to the constraints that the total initial outlay must not exceed ₦620,000, and project A and E are mutually exclusive.

Possible combinations of three projects are:

Projects	Expected NPV (₦)	Total expected NPV (₦)	Total outlay (₦)
A,B,D	6,350+1,882+8,294	16,526	606,000
A,B,F	6,350+1,882+4,993	13,225	576,000
A,D,F	6,350+8,294+4,993	19,637	576,000
B,D,E	1,882+8,294+5,490	15,666	540,000
B,D,F	1,882+8,294+4,993	15,169	510,000
D,E,F	8,294+5,490+4,993	18,777	510,000

The recommend selection is projects A, D, and F which maximises expected total NPV subject to the constraints.

Notes

1. Project C is not considered as it has a negative NPV.
2. Combinations of 2 projects are also possible, but none would have higher expected total NPV. No combination of 4 or more projects is possible.
3. As the money market is efficient any surplus funds invested in the money market will have 0 NPV. Total NPV cannot be increased by investing surplus funds in the money market.

Solution 2 – Equity Valuation & Analysis

2a) $K_E = 5.34 + 0.83(5.32) = 9.76\%$

Dividend payout ratio = 1-b

$$= \frac{D_0}{E_0} = \frac{0.575}{1.837} = 0.313$$

- Justified leading P/E ratio

$$\frac{P_0}{E_1} = \frac{1-b}{r-g} = \frac{0.313}{0.0976 - 0.0818} = 19.8$$

- Justified trailing P/E ratio

$$= \frac{P_0}{E_0} = \frac{(1-b)(1+g)}{r-g}$$

$$= \frac{(0.313)(1.0818)}{0.0976 - 0.0818} = 21.4$$

b) Actual $\dfrac{P}{E} = \dfrac{56.94}{1.837} = 31$

Trailing P/E ratio 21.4

The stock appears to be overvalued.

This can also be confirmed by using Gordon dividend model:

$$P_0 = \frac{D_1}{r-g} = \frac{0.575(1.0818)}{0.0976 - 0.088}$$

$$= ₦39.37 \text{ vs } ₦56.94$$

Question 3 – Equity Valuation & Analysis

3a) * First we determine the asset beta of EHP:

In the absence of beta of debt, we assume debt is risk-free.

$$\beta_A = \frac{\beta_E \times V_E}{V_E + V_D(1-t)}$$

$$= \frac{1.60 \times 40}{40 + 60} = 0.64$$

Notes:

i) Debt/Asset ratio of 0.6 means D/E = 60/40
ii) No tax is given, we assume t = 0.

* Next, we convert the asset beta of EHP to appropriate equity beta of KRG using its
leverage:

$$\beta_E = \beta_A + (\beta_A - \beta_D)\,{}^{V_D}\!/_{V_E}\,(1-t)$$

$$= 0.64 + (0.64 - 0) \left(\frac{20}{80}\right)(1 - 0) = 0.8$$

Note: KRG's Debt/Equity ratio of 0.20 means $\frac{V_D}{V_D} = \frac{20}{80}$

* Using CAPM, the required return on equity of KRG:

$$K_E = R_F + \beta_E(R_m - R_F)$$

$$= 4.4 + 0.80(9) = 11.60\%$$

b) Expected EPS in year 1($E(E_1)$)is:

$E(E_1) = (BVPS_0)(ROE)$

$\quad = 200 \times 0.12$

$E(D_1) = 24(1 - 0.55) = 10.80$

$g = (ROE)(b) = 12 \times 0.55 = 6.6\%$

Implied $K_E = \frac{E(D_1)}{V_E} + g$

$$= \frac{10.80}{180} + 0.066 = 12.60\%$$

The stock has a positive alpha value ($\propto$):

$\propto$ = Expected return - Required return

  = 12.60 - 11.60 = +1

The stock is under-valued because the expected return (12.60%) more than the required return of 11.60. It is trading at a discount.

(Note: Candidates do not need to compute alpha value to arrive at the above conclusion).

c) Residual income method

  * First, we determine the expected residual income in year 1 ($E(R1_1)$):

  $E(R1_1) = BVPS_0 (ROE - K_E)$

      $= 200(0.12 - 0.116) = 0.80$

* Next, we compute the theoretical share price.

The BVPS and the EPS must grow at the sustainable growth rate of 6.6%. In addition, the residual income must grow at the same rate. This is a key point in answering this part of the question!

Theoretical price of the stock (P_0):

$P_0 = BVPS_0 + PV$ of R1

  $= BVPS_0 + \dfrac{R1_1}{K_E - g}$

  $= 200 + \dfrac{0.80}{0.116 - 0.066}$

  $= 200 + 16 = 216$

Note: With a constant growth to infinity, the theoretical price of the stock can be calculated using the following residual model:

$P_0 = B_0 + B_0 \left(\dfrac{ROE - r}{r - g} \right)$, where

B_0 = current book value per share = ₦200

r = cost of equity = 11.6%

Thus $P_0 = 200 + 200 \left(\dfrac{0.12 - 0.116}{0.116 - 0.066} \right) = ₦216$

At ₦180, the stock is trading at a discount. With this approach, one obtains the same result as with the constant growth dividend model:

$P_0 = \dfrac{E(D_1)}{K_E - g}$

  $= \dfrac{10.80}{0.116 - 0.066} = ₦216$

Question 4 – Fixed Income Valuation & Analysis

4. We need to compute the realised total return of each bond.

* Future value of coupons. Since the reinvestment rate is constant, we make use of the future value of annuity:

$$FV = A \left[\frac{(1+r)^n - 1}{r} \right]$$

 − The 30-year bond:

$$FV = 70 \left[\frac{(1.06)^5 - 1}{0.06} \right] = ₦394.60$$

 − The 20-year bond

$$FV = 65 \left[\frac{(1.06)^5 - 1}{0.06} \right] = ₦366.41$$

* Next, we compute the price of each of the bonds at the end of year 5 (i.e. P_5):

 − The 30-year bond.

The maturity of the bond will fall to 25 years, and its yield is forecast to be 8%;

$$P_5 = 70 \times \left[\frac{1 - (1.08)^{-25}}{0.08} \right] + 1{,}000 \left[\frac{1}{1.08} \right]^{25} = 893.25$$

 − The maturity of the 20-year bond will fall to 15 years, and its yield is forecast to be 7.5%

 − $P_5 = 65 \left[\frac{1 - (1.075)^{-15}}{0.075} \right] + 1{,}000 \left[\frac{1}{1.075} \right]^{15} = 911.73$

 − Next, we compute total cash flow at the end of year 5:

	30–year bond	20–year bond
	₦	₦
Reinvested coupons	394.60	366.41
Terminal value (P_5)	893.25	911.73
Total end value (EV)	1,287.85	1,278.14

* Finally, we compute the realised yield (RY)

$$RY = \left(\frac{EV}{P_0} \right)^{\frac{1}{n}} - 1$$

30-year bond: $\left(\frac{1{,}287.85}{867.42}\right)^{\frac{1}{5}} - 1 = 8.22\%$

20-year bond: $\left(\frac{1{,}287.14}{879.50}\right)^{\frac{1}{5}} - 1 = 7.76\%$

Thus, the 30-year bond offers the higher expected yield.

Question 5 – Fixed Income Valuation & Analysis

5 a) Asset cash flows:

Years	Payments
0	0
1	0
2	150,000
3	0
4	150,000
5	0
6	150,000
7	0
8	150,000
9	0
10	150,000

Present value of asset cash flows:

$$PV_A = \frac{150{,}000}{(1+5\%)^2} + \frac{150{,}000}{(1+5\%)^4} + \frac{150{,}000}{1+5\%^6} + \frac{150{,}000}{(1+5\%)^8} + \frac{150{,}000}{(1+5\%)^{10}}$$

$$= 565{,}004.99$$

Alternatively, the present value can be calculated using annuity. However, to use annuity, we need 2-year rate since the cash flows occur every two years.

$$2 - \text{Yearly rate} = (1.05)^2 - 1 = 10.25\%$$

$$PV_A = 150{,}000 \times \left(\frac{1 - (1.1025)^5}{0.1025}\right) = 565{,}004.99$$

Liability cash flow:

Years	Payments
10	1,000,000

Present value of liability cash flow:

$$PV_L = \frac{1{,}000{,}000}{(1+5\%)^{10}} = 613{,}913.25$$

Net surplus $PV_A - PV_L = 565,004.99 - 613,913.23 = -48,908.26$
Hence we have a negative surplus of ₦48,908.26

b) The duration on the assets side is the weighted average of the durations of the individual zero coupon bonds, given as:

$D_A = \sum_{i=1}^{N} x_i . D_i$

where,

$D_A = Duration\ of\ asset\ portfolio$

$D_i = Duration\ of\ individual\ zero - coupon\ bonds\ (2, 4, 6, 8, 10)$

$x_i = Proportion\ of\ wealth\ investment\ in\ bond\ \frac{PV_i}{PV_A}, as\ calculated\ in\ a.\ above$

$$D_A = \frac{1}{565,004.99} (136,054.42.2 + 123,405.37.4 + 111,932.31.6 + 101,525.90.8 + 92,086.99.10) = 5.61 years$$

Again, we can make use of the duration of level annuity.

$$D = \frac{1 + Y}{Y} - \frac{n}{(1 + Y)^n - 1}, where$$

Y = 2-Yearly Yield = 10.25%

$n = number\ of\ the\ receipts = 5$

$$= \frac{1.1025}{0.1025} - \frac{5}{(1.1025)^5 - 1} = 2.8056$$

Since the cash flows occur every two years, we need to multiply this by 2. Thus:
$D_A = 2.8056 \times 2 = 5.61$ Years.

The duration of the liability side is same as its maturity: $D_L = 10$years

c) Yes, generally there will be a change in the net value of asset and liability if there is an upward shift in the term structure of the interest rate.

The approximate change in the value of asset is given by,

$$\Delta P_A = \frac{D_A}{(1 + Y)}. PV_A. \Delta Y$$

$$= -\frac{5.61}{(1 + 5\%)} \times 565,004.99 \times 1\% = -30,187.41$$

On the asset side as the market yield increases by 100 basis points, we tend to loose ₦30,187.41.

The approximate change in the value of liability is given by:

$$\Delta P_L = -\frac{D_L}{(1 + Y)}. PV_L. \Delta Y$$

$$= -\frac{10}{(1 + 5\%)} \times (-613,913.25) \times 1\% = 58,467.93$$

On the liability side as the market yield increases by 100 basis points, we tend to gain ₦58,467,93.

$$\text{Net change} = \Delta P_A - \Delta P_L = -30{,}187.41 + 58{,}467.93 = 28{,}280.52$$

The total gain due to the shift in the term structure of the interest rate is ₦28,280.52. Thus from above we can conclude that if we have a negative surplus, for any upward shift in the term structure of the interest rate, we tend to loose on the asset side and gain on the liability side which results in the net change to be positive.

d) To immunize the surplus, the below equation must hold good:

$$PV_A . D_A = PV_L . D_L$$

Since we can adjust the payback period of mortgage repayment for immunization, the new repayment duration should be:

$$D_L = \frac{PV_A . D_A}{PV_L} = \frac{565{,}004.99 \, . \, 5.61}{613{,}913.25} = 5.16 \text{years}$$

Hence if the market value of your liability remains unchanged, you can be immunized against the shift in the term structure of the interest rate if you repay your mortgage at 5.16 years from today instead of 10 years.

e) No, immunization as done above will not remain forever without rebalancing for the reasons given below:

   i.   the duration of the bond portfolio changes non linearly through time while the duration of the liability decreases linearly with time.

   ii. movements in the interest rate term structure may alter the duration of the bond portfolio.

There is therefore a need to readjust and rebalance the bond portfolio as that the duration of the portfolio matches that of the liability.

Question 1 – Derivatives Valuation & Analysis

1. Consider a stock of company ABC trading in the market at ₦50. Consider a call option on the stock ABC having a exercise price of ₦45 and expiration of 1 year is trading at ₦9 and a corresponding put option is trading at ₦3. Assume that both the options are European, the stock pays no dividends and that the risk free rate of interest prevailing in the economy is 3% (continuously compounded).

   a) Consider that the stock market regulator has prohibited the short selling of company ABC stock; how would you create a synthetic short position of stock using the options and risk free bonds? Assume that put-call parity does hold and that short selling of the options is allowed with no transaction costs.

   b) Does put-call parity hold for the data given above? If not what should be the price of the put option as per put-call parity assuming that the call option is priced correctly? If yes, explain why.

   c) How would a risk free profit be made for the situation in b) above if the put is overpriced? Provide detailed calculation for stock price at expiration both above and below the strike price. Assume no transaction costs.

   d) Will the mispricing of the put option last long? Why? If not, explain the mechanism that will restore the put-call parity back in the market.

(20 marks)

Question 2 – Portfolio Management

2. Given the recent increase of volatility on equity markets, one of your clients is considering the introduction of a portfolio insurance strategy. He is familiar with puts and calls, but not really with portfolio insurance.

   a) Explain briefly to him what portfolio insurance is.

   b) Your client foresees a long period of troubled markets, and asks what would happen if he had to insure his portfolio for several years, knowing that the liquidity on options with a maturity beyond three months is rather poor. Explain to him three rollover strategies, as well as their potential problems.

c) Your client has seen that some banks were offering a Constant Proportion Portfolio Insurance (CPPI) program. However, these programs were not using options. He asks you to explain briefly how these programs work.

d) How would a CPPI strategy perform in a non-directional oscillating market? How about a trending market? **(12 marks)**

Question 3 – Portfolio Management

3. A client wishes to invest in your bank's internal funds that have the following characteristics:

	Expected return	Volatility (Standard deviation)	Correlation coefficient		
			A	B	C
Fund A (Money market)	6%	3%	1		
Fund B (Bonds)	8%	5%	0.7	1	
Fund C (Equities)	12%	18%	0.2	-0.3	1

The simple risk-free rate is 5% p.a.

The client is convinced by your bank's funds and decides to invest 10% in fund A, 40% in fund B and 50% in fund C.

a) Calculate the expected return and the volatility of the portfolio.

b) Explain to your client what is an efficient portfolio and what is an efficient frontier.

c) Let's assume that the answers of question a) give a return and a volatility of 9.0%. Demonstrate to your customer, by combining only 2 funds in a new portfolio, that it is possible to find a more efficient portfolio while keeping an expected return of 9.0% (without leverage).

d) Eventually, your client thinks that the portfolio he has selected is too risky and wants to have a portfolio with a volatility of 6%. Moreover, he wants to invest only in fund C and in the risk-free rate (R_f = 5%). Find the relative weights of the new portfolio invested in these 2 assets as well as the return of this new portfolio. **(16 marks)**

Question 4 – Commodity Trading & Futures

4 a)i) Explain what is meant by a perfect hedge. Does a perfect hedge always lead to a better outcome than an imperfect hedge? Explain your answer.

   ii) Give three reasons why the treasurer of a company might not hedge the company's exposure to a particular risk.

   iii) The standard deviation of monthly changes in the spot price of live cattle is (in cents per pound) 1.2. The standard deviation of monthly changes in the futures price of live cattle for the closest contract is 1.4. The correlation between the futures price changes and the spot changes is 0.7. It is now October 15. A beef producer is committed to purchasing 200,000 pounds of live cattle on November 15. The producer wants to use the December live cattle futures contracts to hedge its risk. Each contract is for the delivery of 40,000 pounds of cattle. What strategy should the beef producer follow?

 b) i) A corn farmer argues "I do not use futures contracts for hedging. My real risk is not the price of corn. It is that my whole crop gets wiped out by the weather". Discuss this viewpoint. Should the farmer estimate his or her expected production of corn and hedge to try to lock in a price for expected production?

   ii) An airline executive has argued: "There is no point in our using oil futures. There is just as much chance that the price of oil in the future will be less than the futures price as there is that it will be greater than this price". Discuss the executive's viewpoint.

   iii) It is now July 2020. A mining company has just discovered a small deposit of gold. It will take 6 months to construct the mine. The gold will then be extracted on a more or less continuous basis for 1 year. Futures contracts on gold are available on the New York Commodity Exchange. There are delivery months every 2 months from August 2020 to December 2021. Each contract is for the delivery of 100 ounces. Discuss how the mining company might use futures markets for hedging.   **(16 marks)**

Question 5 – Commodity Trading & Futures

5. Mr Z holds 1,000 tonnes of copper, with a current price of ₦4,500 per tonne. He writes a single call option (on 1,000 tonnes) with an exercise price of ₦4,750 and for a premium of ₦125 per tonne. What is the outcome for Mr Z if the price of copper at expiry is:

   i)   ₦4,000

   ii) ₦4,600

   iii) ₦5,200   **(6 marks)**

Paper 3.0(B)
Section B (Theory)
Set 1

Solution 1 – Derivatives Valuation & Analysis

1a) From the put-call parity equation we have

$$C + E_e^{-rT} = P + S$$

$Hence$ short stock i. e. synthetic short stock is given by $- S = P - C - E_e^{-rT}$

This means that short stock = long put + short call + short riskless bond.

 b) To check the put-call parity, we should use the equation as given above.

$$P = C - S + E_e^{-rT}$$

Given that the call option is correctly priced, we need to calculate the (synthetic) put price using the above equation to check against the actual price of the traded put option in the market

$$P = C - S + E_e^{-rT} = 9 - 50 + 45e^{-0.03 \times 1} = 2.67$$

Thus, the put option should sell for ₦2.67 whereas it is priced at ₦3. Hence the put-call parity does not hold in this case.

 c) As calculated in (b) above, the traded put is overvalued. To earn a risk-free profit, investors sell the traded put and buy the synthetic put. Buying the synthetic put means:

Long the call

Short the stock

Long (lend) E_e^{-rT}

Cash Flows Today	₦
Short traded put	+ 3.00
Long the traded call	- 9.00
Short the stock	+50.00
Long the bond $-45e^{-0.03 \times 1}$	- 43.67
	0.33

Hence, there will be an initial cash inflow of ₦0.33

Now at the expiration we need to reverse our position. So let's check the payoff at the expiration date.

	If $S_T \leq 45$	If $S_T > 45$
	₦	₦
Short put	- (45- S_T)	0
Long call	0	S_T – 45
Short stock	- ST	- ST
Long bonds	+45	+45
Net cash flow at expiration	0	0

We can see that for given stock price at the expiration date, there will be no cash outflow and hence no risk in holding onto the position. However there was a made at the time of entering contract. So all in all a risk free profit of ₦0.33 is made by adopting the above described strategy.

d) As seen above in c), the overvaluation of the put will help an investor to sell put and buy calls, buy risk free bonds and sell short stock to make a risk free profit. This same strategy will be adopted by other operators in the market. This will create a huge demand for the call option resulting in its price to fall. This will help in the restoration of the put-call parity back in the market wherein the market of risk free profit will cease to exist. Hence in an efficient market the above disparity in the put and the call option prices will not last long.

Solution 2 – Portfolio Management

2 a) Portfolio insurance is a strategy that aims at limiting the downside risk of a portfolio while maintaining its upside potential. Portfolio insurance has traditionally taken two forms: the purchase of put options on the portfolio (static), and the dynamic replication of those same put options (dynamic).

b) The rollover strategies differ primarily with respect to the choice of the new strike price of the options at the time of the rollover:

- The fixed strike strategy maintain the strike price of the put options constant. It is simple to understand, but might result in requiring deeply in or out of the money put options if the portfolio experiences sharp movements.
- The fixed percentage strategy maintains the strike price of the put options equal to a fixed percentage of the underlying portfolio value. If the market keep declining, it does not really offer any downside protection.
- The ratchet strategy starts with a strike price equal to a fixed percentage of the underlying portfolio value and can only increase it or keep it constant at

the time of a rollover. If the market keeps declining, this strategy will offer some downside protection, but may not be able to participate in a recovery ("cash out").

c) CPPI is a dynamic asset allocation strategy that essentially buys more equities when they arise and sells them as they decline. To implement a CPPI strategy, the investor selects a floor below which his portfolio value is not allowed to fall. If we think of the difference between the portfolio and floor as a 'cushion' then the CPPI decision rule is to simply keep the exposure to shares a constant multiple of the cushion.

d) In a flat oscillating market, a CPPI will do relatively poorly, as the investor buys on strength only to see the market weaken, and sells on weakness only to see the market rebound. In a trending bull market, the CPPI strategy will do very well as the investor is buying more shares as they rise. In a trending bear market, the investor will find itself fully in cash at some point.

Solution 3 – Portfolio Management

3 a) $E(R_P) = 0.1.\,0.06 + 0.4\,.\,0.08 + 0.5.\,0.12 = 9.8\%$

Variance $= \sigma_P^2 = 0.1^2.\,3^2 + 0.4^2.\,5^2 + 0.5^2.\,18^2 + (2\,.\,0.1\,.\,0.4\,.\,0.7\,.\,3.5)$

$\qquad +(2\,.\ 0.1\,.\,0.5\,.\,0.2\,.\ 3.18) + (2\,.\,0.4\,.\,0.5\,.\,(-0.3).\,5.18) = 76.21\ (\%)^2$

Therefore, the volatility is $\sigma R_P = (76.21)^{\frac{1}{2}} = 8.73\%$

b) An efficient portfolio is a portfolio that is located on the efficient frontier. The efficient frontier is the result of the combination of a given set of securities where you only select the most relevant portfolios. In this context you (the client) will choose your optimal portfolio so that it:

- offers a minimum risk for a given level of expected return or

- offers a maximum expected return for a given risk level.

c) Let us choose a combination of 2 funds with a negative correlation coefficient: B and C.

To keep a return of 9% we have:

X_b = weight invested in fund B and $(1 - X_b) = X_c$ = weight invested in fund C

$9 = X_b\,.8 + (1 - X_b)\,.12$

$X_b = 0.75$ and $X_c = 1 - 0.75 = 0.25$

(Check: 0.75 .8 + 0.25 .12 = 9% OK)

Now let's calculate the volatility:

$$\sigma_p^2 = 075^2 . 5^2 + 0.25^2 . 18^2 + 2. 0.75. 0.25. (-0.3). 5.18$$

$$= 24.188(\%)^2$$

And $\sigma_P = (24.188)^{1/2} = 4.918\%$

We have demonstrated that we were able, by combining only 2 funds, to keep a return of 9% and to decrease the volatility from 9% to 4.918%.

d) For a portfolio made up of a risk-free asset and a risky asset, the variance is given by:

$\text{Variance} = (Wc^2)(\sigma c^2)$, where

W_C = weight of the risky asset in the portfolio

$\sigma_C{}^2$ = variance of the risky asset

In this case:

$$6^2 = (Wc^2)(18^2)$$

$$Wc^2 = \frac{6^2}{18^2}$$

$$W_c = \frac{1}{3}$$

Thus, $\frac{1}{3}$ of the portfolio value should be invested on fund C and $\frac{2}{3}$ on the risk-free asset. The return is: $E(R_P) = \left(\frac{1}{3} \times 12\right) + \left(\frac{2}{3} \times 5\right) = 7.33\%$

Solution 4 – Commodity Trading & Futures

4 a) i) A perfect hedge is one that completely eliminates the hedger's risk. A perfect hedge does not always lead to a better outcome than an imperfect hedge. It just leads to a more certain outcome. Consider a company that hedges its exposure to the price of an asset. Suppose the asset's price movement prove to be favorable to the company. A perfect hedge totally neutralizes the company's gain from these favorable price movements. An imperfect hedge, which only partially neutralizes the gains, might well give a better outcome.

ii) a) If the company's competitors are not hedging, the treasurer might feel that the company will experience less risk if it does not hedge.

b) The shareholders might not want the company to hedge.

c) If there is a loss on the hedge and a gain from the company's exposure to the underlying asset, the treasurer might feel that he or she will have difficulty justifying the hedging to other executives within the organization.

iii) The optimal hedge ratio is

$$0.7 \times \frac{1.2}{1.4} = 0.6$$

The beef producer requires a long position in 200,000 × 0.6 = 120,000 1bs of cattle. The beef producer should therefore take a long position in 3 December contracts closing out the position on November 15.

b) i) Suppose that the weather is bad and the farmer's production is lower than expected. Other farmers are likely to have been affected similarly. Corn production overall will be low and as a consequence the price of corn will be relatively high. The farmer is likely to be over hedged relative to actual production. The farmer's problems arising from the bad harvest will be made worse by losses on the short futures position. This problem emphasizes the importance of looking at the big picture when hedging. The farmer is correct to question whether hedging price risk while ignoring other risks is a good strategy.

ii) It may well be true that there is just as much chance that the price of oil in the future will be above the futures price as that it will be below the futures price. This means that the use of a futures contract for speculation would be like betting on whether a coin comes up heads or trails. But it might make sense for the airline to use futures for hedging rather than speculation. The futures contract then has the effect of reducing risks. It can be argued that an airline should not expose its shareholders to risks associated with the future price of oil when there are contracts available to hedge the risks.

iii) The mining company can estimates its production on a month by month basis. It can then short futures contracts to lock in the price received for the gold. For example, if a total of 3,000 ounces are expected to be produced in January 2006 and February 2006, the price received for this production can be hedged by shorting a total of 30 February 2006 contracts.

Solution 5 - Commodity

5. i) There is a loss of ₦500 (₦4,500 − ₦4,000), however the option is not exercised, so Mr Z keeps the premium of ₦125, resulting in an overall loss of (₦500 - ₦125) = ₦375 per tonne.

ii) There is a profit of ₦100 and the option is not exercised, so Mr Z keeps the premium of ₦125, resulting in an overall profit of ₦100 + ₦125 = ₦125 per tonne.

iii) The gain on each tonne would have been ₦700 (₦5,200 − ₦4,500), but the option is exercised, so Mr Z needs to deliver the copper at ₦4,750, a gain of only ₦250 (₦4,750 − ₦4,500). However, he also keeps the premium of ₦125, resulting in an overall profit of ₦250 + ₦125 = ₦375 per tonne.

Question 1 – Corporate Finance

1a) Discuss how the capital asset pricing model can be used to calculate a project-specific cost of capital referring in your discussion to the key concepts of systematic risk, business risk and financial risk.   **(3 marks)**

 b) Discuss why the cost of equity is usually greater than the cost of debt.   **(2 marks)**
(Total: 5 marks)

Question 2 – Corporate Finance

2.   Zenco Plc (ZP), a food retailer with 30 superstores spread across Nigeria, is proposing to make a takeover bid for Aladun Ltd (AL), a company with nine superstores.

Extracts from the latest financial statements of both companies are as follows:

	ZP	AL
	₦000	**₦000**
Sales	884,200	93,600
Profit before tax	123,800	15,600
Taxation	24,800	3,100
Profit after tax	99,000	12,500
Dividends	54,000	5,500
Retained earnings for the year	45,000	7,000
Non-current assets	449,000	67,000
Current assets	69,000	4,940
Non-current liabilities	194,000	17,460
Current liabilities	72,000	8,720
Share capital (40 kobo per share)	88,000	10,000
Reserves	164,000	35,760

ZP's current share price is ₦3.60 per share and it has estimated that AL's price to earnings ratio is 12.5% higher than ZP's current price to earnings ratio. ZP's non-current liabilities include a 6% bond redeemable in three years at par which is currently trading at ₦104 per ₦100 par value. ZP estimates that it could achieve synergy savings of 30% of AL's estimated equity value by eliminating duplicated administrative functions, selling excess non-current assets and through reducing the workforce numbers, if the acquisition were successful.

ZP proposes to pay for the acquisition using one of the following three methods.

Method 1
A cash offer of ₦5.00 per AL's share; or

Method 2
An offer of three of its shares for two of AL's shares; or

Method 3
An offer of 2% coupon bond in exchange for 16 AL's shares. The bond will be redeemed in three years at par value of ₦100.

Required
a) Explain briefly, in general terms, why many acquisitions in the real world are not successful **(4 marks)**
b) Estimate the percentage gain on an AL's share under each of the above three payment methods. Comment on the answers obtained. **(13 marks)**
(Total: 17 marks)

Question 3 – Equity Valuation & Analysis

3. You are an analyst with Big Heart (BH) Asset Management company, based in Abuja. You have been following BB Plc, a construction company listed on NSE at a current price of ₦120.00.

    After rigorous consideration of the characteristics of the company and its major competitors, you believe the company will have extraordinary growth for the next few years and normal growth thereafter. You have therefore decided that a two-stage dividend valuation model will be appropriate.

    BB Plc pays semi-annual dividends. The total dividends for the year just ended was ₦2.50.

    You realise that even within the two-stage dividend valuation model, there could be some variations in approach. You will like to explore how these variations affect the valuation of the stock.

    In consultation with your supervisor, the following approaches are to be considered.

    i) The dividend growth rate will be $12^1/_2$ throughout the first stage of ten years. The dividend growth thereafter will be 6%.

    ii) In contrast to the first approach in which the growth rate of 6% in the second stage, you want to use your estimate that ten years later BB stock will be worth 20 times its EPS (trailing P/E of 20). You expect earnings retention rate of 75%.

You have assembled the following facts about the company and the stock market generally:

Debt/equity ratio of BB	1:1
Asset beta of BB	0.8
Debt beta of BB	0.2
Effective tax rate	25%
Equity risk premium	6%
Risk-free rate	2.5%
Cost of debt, before tax of BB	4%

You are required to answer the following questions:

a) List 3 conditions that make dividend valuation model (rather than free cash flow models, for example most suitable for equity valuation). **(2 marks)**

b) Justify and calculate the relevant required return you will use in this valuation. Show all workings. **(3 marks)**

Note: In subsequent parts of this question where rate of return is needed and you are unable to compute it as required in (b) above, assume a rate of 11%.

c)i) Compute the terminal value of the stock based on the first approach. **(2 marks)**

ii) What is the current value of the stock, based on the first approach? **(1 mark)**

d) Based on the second approach, what is the current value of the stock? **(2 marks)**

e) In this part of the question, assume the required return is 12%. Assume further the stock is fairly priced at ₦120. Hold other estimates constant.

i) What perpetual growth rate in dividend is implied by the price of ₦120? **(2 marks)**

ii) What dividend yield is implied by the price? **(2 marks)**

Note: If you need the result in (e)(i) and which you could not calculate, assume growth rate of 9.50%.

(Total: 14 marks)

Question 4 – Equity Valuation & Analysis

As a firm operating in a mature industry, Akoka Industries is expected to maintain a constant dividend payout ratio and constant growth rate of earnings for the foreseeable future. Earnings were ₦4.50 per share in the recently completed fiscal year. The dividend payout ratio has been a constant 55% in recent years and is expected to remain so. Akoka's return on equity (ROE) is expected to remain at 10% in the future, and you require an 11% return on the stock.

a) Using the constant-growth dividend valuation model, calculate the current value of Akoka shares. Show your calculations.

   After an aggressive acquisition and marketing program, it now appears that Akoka's earnings per share and ROE will grow rapidly over the next two years. You are aware that the dividend valuation model can be useful in estimating the value of the shares even when the assumption of constant growth does not apply.　　(**6 marks)**

b) Calculate the current value of Akoka's shares, using the dividend valuation model, assuming that Akoka's dividend will grow at a 15% rate for the next two years, returning in the third year to the historical growth rate and continuing to grow at the historical rate for the foreseeable future. Show your calculations.　　(**6 marks)**

(Total: 12 marks)

Question 5 – Fixed Income Valuation & Analysis

5a) i) Describe each of the two prepayment risks for a mortgage pass-through security and relate each risk to changes in interest rates.　　(**2 marks)**

   ii) Determine whether the value of a callable convertible bond will increase, decrease, or remain unchanged in response to each of the following changes, and justify each of your responses with one reason:

   - An increase in stock price volatility
   - An increase in interest rate volatility　　(**2 marks)**

 b) Assume that the 1-year spot rate is 4.5749%. Also assume that the 1-year forward rate is either 5.3210% or 7.1826%.

   i) Using the backward induction methodology, calculate the value of a 10% annual-pay-option-free bond with a Z – year maturity.　　(**6 marks)**

   ii) Assume the bond in (i) is currently callable after one year at 103 and that 50 basis points should be added to each forward rate to reflect the call feature. Using backward induction, calculate the value of the callable bond.　　(**6 marks)**

   iii) In (i) above, assume the bond is putable after one year at 100, but is **not** callable at 103 as in (ii). Calculate the value of the putable bond.　　(**6 marks)**

(Total: 22 marks)

Solution 1 – Corporate Finance

1a)   The capital asset pricing model (CAPM) assumes that investors hold diversified portfolios, so that unsystematic risk has been diversified away. Companies using the CAPM to calculate a project-specific discount rate are therefore concerned only with determining the minimum return that must be generated by an investment project as compensation for its systematic risk.

The CAPM is useful where the business risk of an investment project is different from the business risk of the investing company's existing business operations. In such a situation, one or more proxy companies are identified that have similar business risk to the investment project. The equity beta of the proxy company represents the systematic risk of the proxy company, and reflects both the business risk of the proxy company's business operations and the financial risk arising from the proxy company's capital structure.

Since the investing company is only interested in the business risk of the proxy company, the proxy company's equity beta is 'ungeared' to remove the effect of its capital structure. 'Ungearing' converts the proxy company's equity beta into an asset beta, which represents business risk alone. The asset betas of several proxy companies can be averaged in order to remove any small differences in business operations.

The asset beta can then be 'regeared', giving an equity beta whose systematic risk takes account of the financial risk of the investing company as well as the business risk of an investment project. Both ungearing and regearing use the weighted average beta formula, which equates the asset beta with the weighted average of the equity beta and the debt beta.

The project-specific equity beta resulting from the regearing process can then be used to calculate a project-specific cost of equity using the CAPM. This can be used as the discount rate when evaluating the investment project with a discounted cash (DCF) flow investment appraisal method such as net present value or internal rate of return. Alternatively, the project-specific cost of equity

can be used in calculating a project-specific weighted average cost of capital, which can also be used in a DCF evaluation.

b) The cost of equity is the return required by ordinary shareholders (equity investors), in order to compensate them for the risk associated with their equity investment, i.e. their investment in the ordinary shares of a company. If the risk of an investment increases, the return expected by the investor also increases. If the risk of a company increases, therefore, its cost of equity also increases.

If a company is liquidated, the order in which the claims of creditors are settled is a factor in determining their relative risk.
The claims of providers of debt finance (debt holders) must be paid off before any cash can be distributed to ordinary shareholders (the owners). The risk faced by shareholders is therefore greater than the risk faced by debt holders, and the cost of equity is therefore greater than the cost of debt.

Interest on debt finance must be paid before dividends can be paid to ordinary shareholders, so the risk faced by ordinary shareholders is greater than the risk faced by debt holders, since the necessity of paying interest may mean that dividends have to be reduced.

Solution 2 – Corporate Finance

2a) **Common reason why acquisitions are unsuccessful**
Lack of industrial or commercial fit
Failure can result from a takeover where the acquired entity turns out not to have the product range or industrial position that the acquirer anticipated.
Lack of goal congruence
This may apply not only to the acquired entity but, more dangerously, to the acquirer, whereby disputes over the treatment of the acquired entity might well take away the benefits of an otherwise excellent acquisition.
'Cheap' purchases
The 'turn around' costs of an acquisition purchased at what seems to be a bargain price may well turn out to be a high multiple of that price.
Paying too much
The fact that a high premium is paid for an acquisition does not necessarily mean that it will fall. Failure would result only if the price paid is beyond that which the acquirer considers acceptable to increase satisfactorily the long term wealth of its shareholders.

Failure to integrate effectively

An acquirer needs to have a workable and clear plan of the extent to which the acquired company is to be integrated. The plan must address such problems as differences in management styles, incompatibilities in data information systems, and continued opposition to the acquisition by some of the acquired entity's staff.

b) Number of shares in ZP:

ZP EPS = ₦99,000,000/220,000,000 = ₦0.45

ZP PE ratio = ₦3.6/0.45   = 8

AL PE ratio = 8 × 1.125   = 9

AL number of shares:

AL EPS = 12,500,000/25,000,000   = ₦0.50

AL VPS = ₦0.50 × 9   = ₦4.50

Cash Offer

AL percentage gain = (₦5/₦4)-1   =11.1%

Share – for – share exchange	**₦000**
Equity value of ZP = 20,000,000 × ₦3.60	= 792,000
Equity value of AL = 25,000,000 × ₦4.50	= 112,500
Synergy savings = 30% × 112,500,000	= 33,750
Total equity value of combined company	= 938,250
Number of shares:	
- Currently in ZP (000)	= 220,000
- Issued for AL: 25,000,000 × 3/2	= 37,500
Revised number of shares in ZP	= 257,500

Expected VPS = 938,250/257,500 = ₦3.64/share. With a share exchange ratio of 3/2 = 1.5, it means every share in AL will be worth 1.5 × ₦3.64 = ₦5.46 in ZP after the take takeover.

Percentage gain = (₦5.46/4.5)-1 = 21.33%.

Bond offer

Step 1: Determine the YTM of ZP's current bond.

Trying both 4% and 5%

Year	CF	PV at 4%	PV at 5%
0	(104)	(104)	(104)
1 – 3	6	16.651	16.339
3	100	88.90	86.384
NPV		1.551	- 1.277

$$\text{YTM} = \text{IRR} = 4 \times \frac{1.551}{1.551 \times 1.277}(5-4) = 4.55\%$$

On the assumption that the bond to be issued is of the same rating as the existing bonds of ZP, we can make use of the YTM of 4.55% to price the bond.

Step 2: Calculate the price of the bond.

$$P = \frac{2}{4.055} + \left(\frac{2}{4.055}\right)^2 \times \left(\frac{102}{4.055}\right)^3 = ₦93$$

Value per share = ₦93/16 = ₦5.81
Percentage gain = (₦5.81/4.50)-1 = 29.1%

Comment:
An initial comparison is made between the cash and the share-for-share offers. Although the share-for-share exchange gives a higher return compared to the cash offer, AL'S shareholders may prefer the cash offer as the gains in the share price are dependent on the synergy gains being achieved. However, purchase for cash may mean that the shareholders face an immediate tax burden. ZP's shareholders would probably prefer the cash option because the premium would only take ₦12,500,000 of the synergy benefits (₦0.50 × 25,000,000 shares), whereas a share-for-share exchange would result in approximately ₦23,996,250 of the synergy benefits being given to the AL's shareholders (21.33% × ₦4.50 × 25,000,000 shares).

The bond offer provides an alternative which may be acceptable to both sets of shareholders. AL's shareholders receive the highest return for this and ZP's shareholders may be pleased that a large proportion of the payment is deferred for three years. In present value terms, however, a very high proportion of the projected synergy benefits are given to AL's shareholders (29.1% × ₦4.50 × 25,000,000 = ₦32,737,500).

Solution 3 – Equity Valuation & Analysis

3a) The 3 conditions are:

- The company dividend is paying i.e the analyst has dividend records to analyse;
- The investor takes a noncontrol perspective; and
- The directors have established a dividend policy that bears and understandable and consistent relationship to company's profitability.

b) Since we are pricing equity using dividend valuation model, the appropriate rate of return to use is cost of equity. First, we compute beta of equity:

$$\beta_E = \beta_A + (\beta_A - \beta_D)\left(\frac{D}{E}\right)(1 - t)$$

$$= 0.8 + (0.8 - 0.2)\left(\frac{1}{1}\right)(1 - 0.25) = 1.25$$

Next, we compute the required return, r, using CAPM.

r = 2.5 + 1.25 (6) =10

c) i) $TV_{10} = \dfrac{D_{11}}{r - g}$

$$D_{11} = 2.50\ (1.125)^{10}(1.06) = 8.605$$

$$TV_{10} = \frac{8.605}{0.10 - 0.06} = 215.125$$

ii) First, compute the PV of dividends for the next 10 years, when g = 12.5%. This can be calculated fastly, using growing annuity.

$$PV = \frac{D_0(1 + g)}{r - g}\left[1 - \left(\frac{1 + g}{1 + r}\right)^n\right]$$

$$= \frac{2.50(1.125)}{0.10 - 0.125}\left[1 - \left(\frac{1.125}{1.10}\right)^{10}\right] = 28.35$$

Add the PV of the terminal value:

$$215.125 \times (1.10)^{-10} = \underline{82.94}$$

$$\text{Total} = P_0 \qquad = \underline{111.29}$$

The PV of dividends for the first 10 years could also have been computed year by year as follows:

$$PV = \frac{2.50(1.125)}{1.10} + \frac{2.50(1.125)^2}{(1.10)^2} + \frac{2.50(1.125)^9}{(1.10)^9} + \frac{2.50(1.125)^{10}}{(1.10)^{10}} = ₦28.35$$

d) First, we compute the terminal value in year 10 (TV_{10})

$D_{10} = 2.50(1.125)^{10} = 8.12$

$E_{10} = 8.12 \times \dfrac{100}{75} = 10.83$

$\dfrac{P}{E} = 20$

$TV_{10} = 20 \times 10.83 = 216.60$

The stock price is then computed:

	₦
PV of dividends for the first 10 years (see above)	28.35
PV of terminal value:	
$216.60 \times (1.10)^{-10}$ =	_83.51_
Total = P_0 =	_111.86_

Note: if a discount rate of 11% is used, the results in (c) and (d) will be affected as follows:

$c\ i)\ TV_{10} = \dfrac{8.605}{0.11 - 0.06} = ₦172.10$

ii) Yrs 1-10 PV of dividends:

$\dfrac{2.50(1.125)}{0.11 - 0.125}\left[1 - \left(\dfrac{1.125}{1.11}\right)^{10}\right]$ = 26.94

Add: PV of TV = $172.10(1.11)^{-10}$ = _60.61_

P_0 = _87.55_

$d)\ P_0 = 26.94 + 216.60(1.11)^{-10}$

$= 26.94 + 76.28$ = _₦103.22_

e) i) We know that:

$P_0 = \dfrac{D_0(1 + g)}{r - g}$

$120 = \dfrac{2.50(1 + g)}{0.12 - g}$

120(0.12 - g) = 2.50(1+g)

48(0.12 - g) = 1 + g

g = 8.07%

ii) Total return of 12% is made up of:

Capital appreciation × Dividend
yield (g)   yield (DY)

Thus:

12 = 8.07 + DY

DY = 12 − 8.07 = 3.9%

Question 4 – Equity Valuation & Analysis

4 a) **According to the Gordon model, the growth rate is determined as follows: g = rb**

where

r   =   return on capital employed = return on equity (ROE) in a pure equity financed company,

b   =   retention rate

         =   100% - Payout rate

         = $\dfrac{\text{PAT} - \text{Dividends}}{\text{PAT}}$

(PAT   =   Profit after tax)

∴ g   =   (10) (1 − 0.55) = 4.50%

Note that current dividend is ₦4.50 (0.55) =₦2.475

The current market value is calculated using the usual formula:

$$PV = \frac{A}{r-g} = \frac{2.475(1.045)}{0.11 - 0.045} = ₦39.79$$

b) i)   **First 2 years when g = 15% using**

$$PV = \frac{A}{r-g}\left(1 - \left(\frac{1+g^n}{1+r}\right)\right)$$

$$\frac{2.475(1.15)}{0.11 - 0.15}\left[1 - \left(\frac{1.15}{1.11}\right)^2\right] \qquad = ₦5.22$$

ii)   **Years 3 to infinity**

Using PV = $\dfrac{A\ (1+r)^{-n+1}}{r-g}$

A   = $2.475(1.15)^2(1.045)$ = 3.42

$$\therefore PV \quad = \frac{3.42}{.11 - .045}(1.11)^{-3+1} = ₦42.70$$

Total value = ₦5.22 + ₦42.70   =₦47.92.

Solution 5 – Fixed Income Valuation & Analysis

5 a) i) The prepayment risk associated with declining interest rates is contraction risk: The upside price potential is compressed because of accelerating prepayments, and the cash flows must be reinvested at lower rates. The average life of the pass-through shortens.

The prepayment risk associated with rising interest rates is extension risk: The price decline is exacerbated because of slowing prepayments. The average life of the pass-through lengthens.

ii)
- The value of the convertible bond = value of straight bond + value of the conversion option + value of the call option on the bond.
- An increase in stock price volatility should increase the value of the convertible. This is because the conversion option becomes more valuable.
- An increase in interest rate volatility should decrease the value of the convertible. This is because the chance of the bond being called increases, causing the value of the call option on the bond to become more valuable.

b) i) The binomial interest-rate tree should look like this:
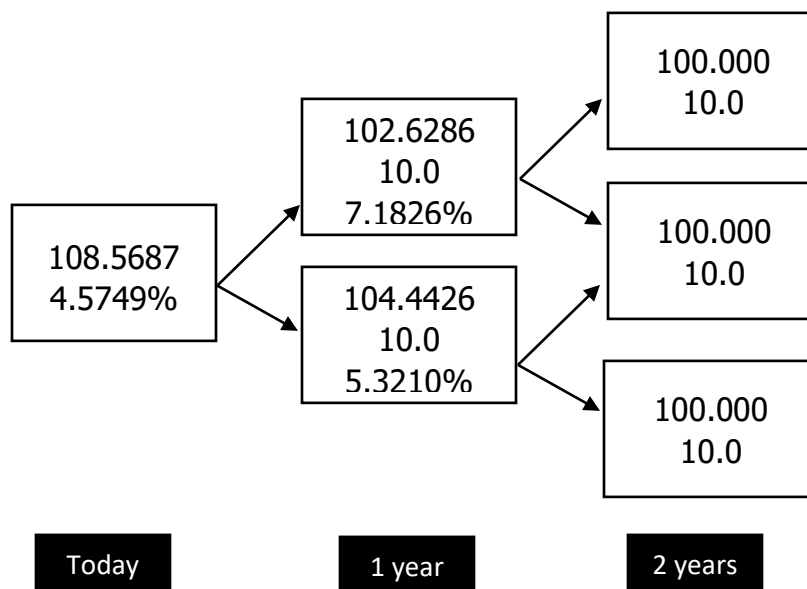
The value of the bond at the upper node for period 1, $V_{1,U}$, is:

$$V_{1,U} = \frac{1}{2}\left[\frac{(100+10)}{1.071826} + \frac{(100+10)}{1.071826}\right] = 102.6286$$

Similarly, the value of the bond at the lower node for period 1, $V_{1,L}$, is:

$$V_{1,L} = \frac{1}{2}\left[\frac{(100+10)}{1.053210} + \frac{(100+10)}{1.053210}\right] = 104.4426$$

V_0, the current value of the bond at node 0, is:

$$V_0 = \frac{1}{2}\left[\frac{(102.6286+10)}{1.045749} + \frac{(104.4426+10)}{1.045749}\right] = 108.5687$$

ii) The binomial interest-rate tree should look like this (be sure to add 50 basis points to each interest rate):

The value of the bond at the upper node for period 1, $V_{1,U}$, is:

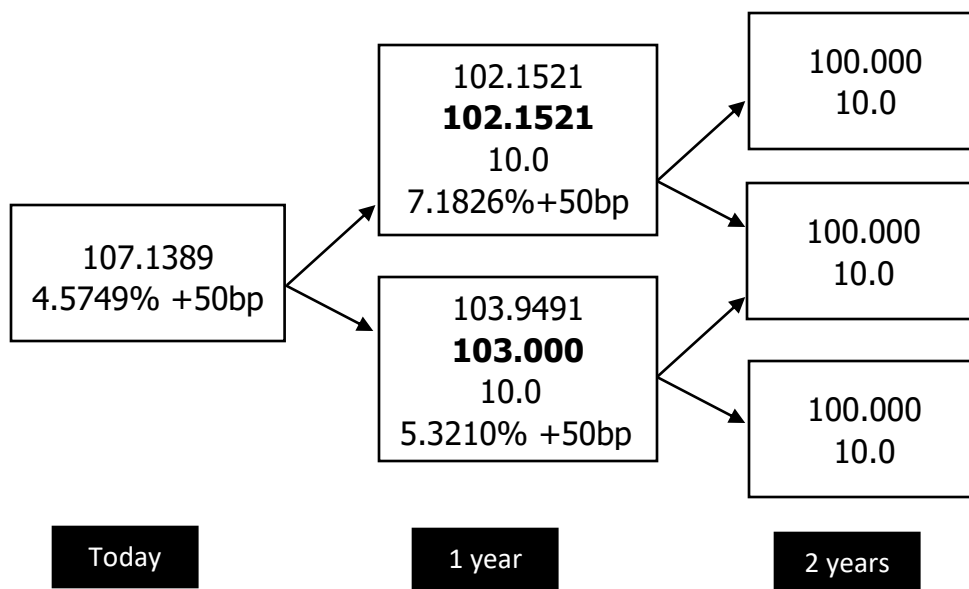$$V_{1,U} = \frac{1}{2}\left[\frac{(100+10)}{1.076826} + \frac{(100+10)}{1.076826}\right] = 102.1521$$

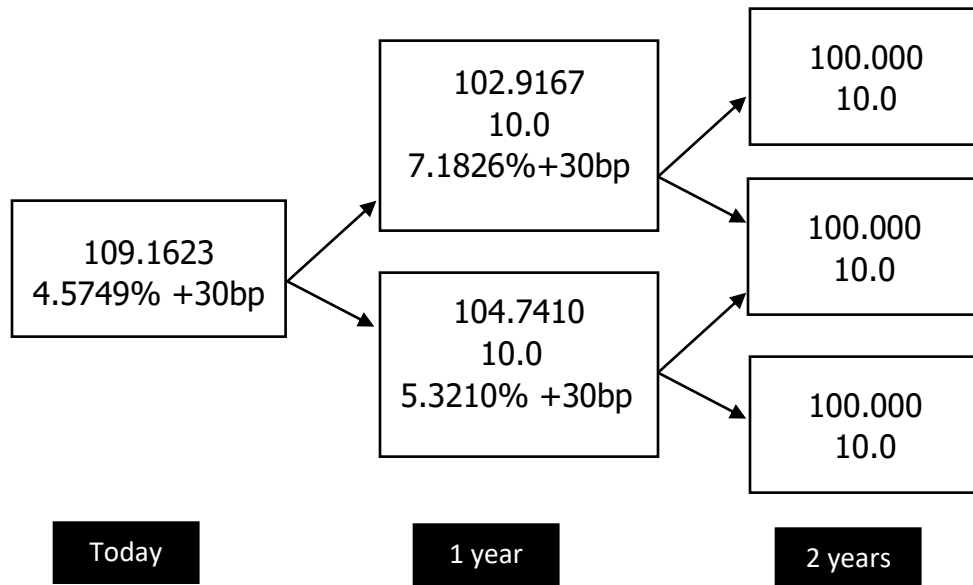Similarly, the value of the bond at the lower node for period 1, $V_{1,L}$, is:

$$V_{1,L} = \frac{1}{2}\left[\frac{(100+10)}{1.058210} + \frac{(100+10)}{1.058210}\right] = 103.9491$$

However, since the bond is callable at 103, replace the value at the lower node (103.9491) with 103.

Now calculate V_0, the current value of the bond at node 0:

$$V_0 = \frac{1}{2}\left[\frac{(102.1521 + 10)}{1.050749} + \frac{(103.0000 + 10)}{1.050749}\right] = 107.1389$$

iii) The tree should look like the following (the put option will not be exercised, as the price is well above 100; be sure to subtract 30 basis points from each interest rate).

		100.000 10.0
	102.9167 10.0 7.1826%+30bp	
		100.000 10.0
109.1623 4.5749% +30bp		
	104.7410 10.0 5.3210% +30bp	
		100.000 10.0
Today	**1 year**	**2 years**

The value of the bond at the upper node for period 1, $V_{1,U}$, is:

$$V_{1,U} = \frac{1}{2}\left[\frac{(100 + 10)}{1.068826} + \frac{(100 + 10)}{1.068826}\right] = 102.9167$$

Similarly, the value of the bond at the lower node for period 1, $V_{1,L}$, is:

$$V_{1,L} = \frac{1}{2}\left[\frac{(100 + 10)}{1.050210} + \frac{(100 + 10)}{1.050210}\right] = 104.7410$$

V_0, the current value of the bond at node 0, is:

$$V_0 = \frac{1}{2}\left[\frac{(102.9167 + 10)}{1.042749} + \frac{(104.7410 + 10)}{1.042749}\right] = 109.1623$$

Paper 3.0(B)
Section B (Theory)
Set 2

Question 1 – Derivatives Valuation & Analysis

1. An asset manager plans to purchase a particular stock in 300 days. The stock is currently selling at ₦175, which pays quarterly dividend according to the following schedule.

Days to ex-Dividend Date	Dividends (₦)
15	2.25
107	2.25
198	2.25
289	2.25

Risk-free rate is 5%, continuously compounded and yield curve is flat. The manager decides to commit to a future purchase of the stock by going long a forward contract on the stock.

a) What is the no-arbitrage price of the contract?

b) Suppose the manager enters into the contract at the price you found in (a) above.
Now, 150 days later, the stock price is ₦180. Determine the value of the forward contract at this point.

c) It is now the expiration day, and the stock price is ₦171.50. Determine the value of the forward contract at this time.   **(20 marks)**

Question 2 – Portfolio Management

2. Olowo is a client of Kudi Asset Management. He started his portfolio with an investment of ₦10,000 on 1 January 2018. The value of his portfolio on 31 December 2018 was ₦13,050. Impressed with the performance, he invested a further amount of ₦10,000 on 1 January 2019 with the firm. On 31 December 2019, he withdrew ₦5,000. The value of his portfolio before withdrawal was ₦25,000. The value at the end of 31 December 2020 was ₦22,500.

a) Calculate the total TWR and the annualized TWR for the portfolio.

b) Calculate the total MWR over the holding period for the portfolio, using the Modified Dietz Method. **(10 marks)**

Question 3 – Portfolio Management

3. The following information is available for three mutual funds.

	R_P	β_P	σ_P	R^2
Fund A	7.0%	0.98	17.4%	0.89
Fund B	6.0%	1.03	19.2%	0.95
Fund C	2.5%	0.04	13.5%	0.17

The return of the market portfolio is 8.0% for the considered period, and the return of risk-free assets is 1.0%.

a) Compute the Sharpe ratios of the three funds for the considered period. What would your ranking of the funds be?

b) Compute the Jensen alphas of the three funds for the considered period. What would your ranking of the funds be? If it differs from the recommendation you have obtained in a), how can you explain that difference? If not, why is it the same?

c) Which fund would you suggest for an investor having only cash in his portfolio and wishing to purchase shares of only one fund? Justify your answer. Would it necessarily remain the same if the investor holds already a diversified portfolio on this market and wishes to add the fund shares in it? **(10 marks)**

Question 4 – Portfolio Management

4. Discuss the advantages and disadvantages for investors of using institutionalized asset management services rather than investing directly. **(8 marks)**

Question 5 – Commodity Trading & Futures

5 a) i) In the futures market, what factors determine whether a particular strategy is a hedge or a speculative strategy?

ii) How are spread and arbitrage strategies forms of speculations? How can they be interpreted as hedges?

iii) Explain the differences among the three means of terminating a futures contracts: an offsetting trade, cash settlement, and delivery. How is a forward contract terminated?

b) List and briefly explain the important contributions provided by futures exchanges.

c) You are a dealer in kryptonite and you are contemplating a trade in forward contract. You observe that the current spot price per ounce of kryptonite is ₦180.00, the forward price for delivery in one year is ₦205.20 and annual carrying costs of metal are 4% of the current spot price. Assume there is no convenience yield for Kryptonite.

i) Calculate the annual return on a riskless zero-coupon security implied by the law of one price.

ii) Describe a trading strategy that would generate arbitrage profits for you if annual return on the riskless security is 5%. What is your arbitrage profit per ounce of Kryptonite? **(22 marks)**

Paper 3.0(B)
Section B (Theory)
Set 2

Solution 1 – Derivatives Valuation & Analysis

1a) **Step 1**: Determine the PV of the future dividends

$$PV = 2.25e^{-0.05 \times 15/365} \qquad = 2.2454$$

$$+$$

$$2.25e^{-0.05 \times 107/365} \qquad = 2.2173$$

$$+$$

$$2.25e^{-0.05 \times 198/365} \qquad = 2.1898$$

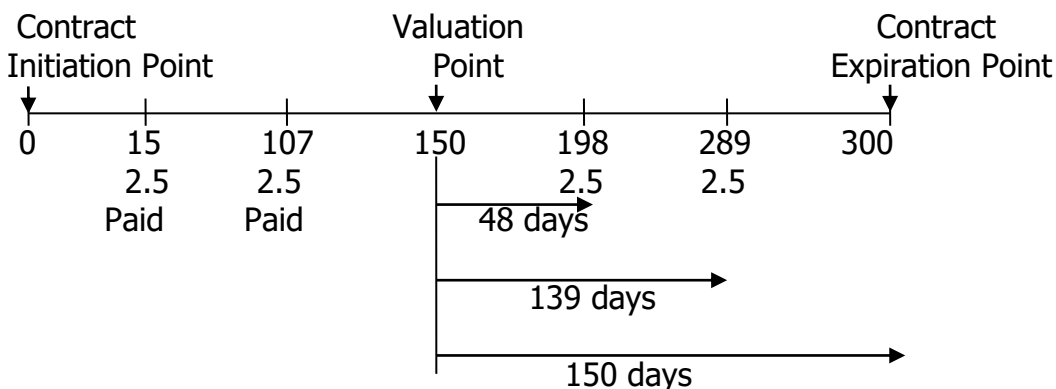$$+$$

$$2.25e^{-0.05 \times 289/365} \qquad = \underline{2.1627}$$

Total PV $\qquad\qquad \underline{8.82*}$

(* The PVs were computed electronically, rounding up only the final answer)

Step 2: Compute the forward price

$$F = (S_0 - 1)\,e^{rT} = (175 - 8.82)e^{(0.05)(300/355)} = ₦173.15$$

b) To value the contract at time t (where t = 150/365), have a careful look at the diagram below:

| Contract
Initiation Point | | | Valuation
Point | | | Contract
Expiration Point |

```
Contract                    Valuation                           Contract
Initiation Point            Point                               Expiration Point
 ↓                           ↓                                   ↓
 |     |       |             |       |       |         |
 0    15      107           150     198     289        300
      2.5     2.5                   2.5     2.5
      Paid    Paid          |  48 days →
                            |       139 days          →
                            |            150 days              →
```

Step 1: Determine the PV of the outstanding dividends, that is, the value as at day 150.

$$PV = 2.5e^{-0.05 \times 48/365} = 2.2353$$

$$+$$

$$= 2.5e^{-0.05 \times 139/365} = \underline{2.2076}$$

Total PV of future cash flows $= \underline{4.44}$

Step 2: Determine the PV of the forward contract.

$$PV(F) = Fe^{-r(T-t)}$$

Remember that:

$$T = 300/365$$

$$t = 150/365$$

$$T - t = (300 - 150)/365 = 150/365$$

Thus:

$$PV(F) = 173.15e^{-0.05 \times 150/365} = 169.628$$

Step 3: Value the contract at time t.

$$V_t = S_t - PV(\text{Future CFs}) - PV(F)$$

$$= 180 - 4.44 - 169.63 = ₦5.93$$

Because this is a positive figure, it is a gain to the long and a loss to the short.

c) To value the contract expiration, we use Equation (3.16).

$$V_T = S_T - F = ₦171.50 - 173.15 = -₦1.65$$

This is a loss to the long.

Solution 2 – Portfolio Management

2a) We have to calculate the compound return from 1 January 2012 to 31 December 2014. The period has three sub-periods:

i) from 1 January 2018 till 31 December 2018.

ii) from 1 January 2019 to 31 December 2019, and

iii) from 1 January 2020 to 31 December 2020.

Sub-period rates of return are calculated as follows:

$$\text{Period 1: } \frac{13{,}050 - 10{,}000}{10{,}000} = 30.50\%$$

Period 2: $\dfrac{25{,}000 - (13{,}050 + 10{,}000)}{13{,}050 + 10{,}000} = 8.46\%$

At the start of period 3, there is a deduction of ₦5,000 so beginning portfolio value for period 3 should be ₦20,000 (₦25,000 − 5,000).

Return for period 3 is:

$\dfrac{22{,}500 - 20{,}000}{20{,}000} = 12.5\%$

The sub-period returns are then chain-linked to determine the total TWR for the whole period.

$[(1.305) \times (1.0846) \times (1.125) - 1] = 59.23\%$

The annualized return is:

$(1.5923)^{1/3} - 1 = 16.77\%$

b) Profit = 22,500 − 10,000 − (10,000 − 5,000) = 7,500

$W_i = \dfrac{CD - D_i}{CD}$

$F_1 = \dfrac{3-1}{3} \times 10{,}000 \qquad\qquad = \qquad 6{,}667$

$F_2 = \dfrac{3-2}{3} \times (-5{,}000) \qquad = \qquad \underline{(1{,}667)}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \underline{5{,}000}$

$R_{MDietz} = \dfrac{7{,}500}{10{,}000 + 5{,}000} \qquad = \qquad 50\%$

3 – Portfolio Management

3a) The Sharpe ratio is defined as

$$SR_P = \frac{R_P - R_F}{\sigma_P}$$

that is, the excess return of the fund divided by the total risk (volatility) of the fund. For out three funds, the ratios are

$$SR_A = \frac{0.07 - 0.01}{0.174} = 0.34$$

$$SR_B = \frac{0.06 - 0.01}{0.192} = 0.26$$

$$SR_C = \frac{0.025 - 0.01}{0.135} = 0.11$$

From that, we can say that the funds A is one that offers the best reward per unit of total risk.

b) The Jensen's α can be interpreted as the market-risk adjusted excess return (or as the realized return in excess to the CAPM predicted return).

$$\alpha_P = R_P - \left(R_F + \beta_P \cdot (R_M - R_F)\right)$$

$$\alpha_A = 0.07 - \left(0.01 + 0.98 \cdot (0.08 - 0.01)\right) = -0.86\%$$

$$\alpha_B = 0.06 - \left(0.01 + 1.03 \cdot (0.08 - 0.01)\right) = -2.21\%$$

$$\alpha_C = 0.025 - \left(0.01 + 0.04 \cdot (0.08 - 0.01)\right) = 1.22\%$$

From this result, we can say that the fund C is the one that performs better according to the CAPM expectations.

Results differ from a) because the risk measure (implicit in the performance measure) is different. Using the Sharpe ratio, we define risk as the total variability (volatility) of the returns. Using the Jensen alpha, we only consider the systematic risk exposure, through the use of the beta coefficient.

c) If an investor owns only cash and wants to invest in one fund, the adequate risk measure is the total volatility of his portfolio, which will result from the inclusion of one risky asset (the fund). Therefore, we should use the Sharpe ratio as a performance measure and select fund A.

If an investor owns already a diversified portfolio, the volatility of the fund is not a relevant risk measure, as an important part of it – the specific risk – will be diversified away when adding the fund in the investor's portfolio. Therefore, using the Sharpe ratio would provide irrelevant results.

Solution 4 – Portfolio Management

4) Investors are inclined to entrust the management of capital to institutions such as investment companies, pension funds and hedge funds for a number of reasons:

- Institutions acquire information about the risks and returns of a huge array of investment opportunities across different markets and are therefore in a better position to deliver these opportunities to investors, and to advise on the associated uncertainties. In the absence of institutional arrangements, investors would lack much of the essential information needed to make judgments on many types of investment.

- Institutions offer knowledge of the investment opportunities and trading customs in different markets. This has become a more significant consideration given that capital markets worldwide have tended to become more open.

- Institutions offer expertise in dealing with legal and regulatory issues. They therefore help to overcome many technical obstacles to security trading and investment that most individuals would find insurmountable.

- By pooling capital from many individuals, investment institutions reduce the transaction costs per investor (brokerage, settlement, money transfer and custody) since it is much cheaper, relatively speaking, to trade securities in large volumes. Savings also arise in other ways. For instance, funds with an overseas investment focus can manage the related currency risk issues more effectively and economically than would be possible for individual investors (wholesale vs retail exchange rates, use of risk management tools).

- Pooling also makes spreading risk more economically viable than it would be for an individual with limited resources. It thereby alleviates the exposure to non-systematic risk for each investor.

- Institutions find it easier and cheaper to construct investments with specific risk-return profiles by resort to leverage and derivatives. For example, investors seeking a leveraged exposure are likely to find the personal cost of borrowing significantly greater than the costs levied on institutions.

- Institutions can more readily securitise investments into tradable claims, thereby offering investors more flexible methods of adjusting asset portfolios in accordance with changing views or needs.

Institutionalised asset management implies a great deal of trust on the part of investors in financial institutions. Disadvantages include:

- Criminal behaviour on the part of fund managers; managers secretly using investors' contributions for purposes other than those claimed. There have been high profile cases (Madoff being the most famous in recent years).

- Opacity of some investment products. For instance, structured investments are marketed as being similar to savings accounts but they involve the use of derivatives, embody counterparty risk and do not guarantee capital.

- Opacity also occurs in relation to operating costs. In the fund management industry there has been a shift from highlighting annual management charges towards a focus on the total expense ratio as a truer measure of the costs of institutional fund management.

- Misrepresentation of risk. For instance, index tracker-type investments (ETFs and mutual fund trackers) are marketed as the safer alternative to active funds.

- However, market (systematic) risk is itself considerable. The gyrations in global stock markets of recent years testify to this.

- Mechanisms of investor protection include: (i) government laws and regulations regarding the disclosure of information, professional behaviour, product marketing and criminality (examples include fines for mis-selling, pressure on banks to increase capital reserves, and convictions of fund managers), (ii) compliance codes and practices within institutions, (iii) codes of professional and ethical practice articulated by self-regulatory organisations (for example, the ICMA), and (iv) media focus on hidden risks and unwarranted claims.

Solution 5 – Commodity Trading & Futures

5 a i) A position in the futures market is accompanied by an opposite position in the spot market, the transaction is a hedge. The hedger does not necessarily have to have a long or short position in the spot market. A hedge can be established if the hedger is reasonably certain of taking a future position in the spot market. The hedge protects against price changes in the interim period until the spot transaction is made. A speculative strategy is not normally accompanied by a transaction or contemplated transaction in the spot market.

ii) A spread strategy is a long position in one futures contract and a short position in another futures contract. The prices of the two contracts are normally highly correlated so that the gains on one contract are at least partially offset by the losses on the other. The objective is to take a small amount of risk in the hope of a small profit. An arbitrage strategy involves a near riskless transaction in one or more futures contracts and possibly a spot transaction. Arbitrage trading is usually triggered by a deviation from the theoretical relationship between the prices of two instruments. Both transactions can be viewed as hedges. A hedge is a position in the spot market and an opposite position in the futures market. Thus, it is similar to a spread in that the gain on one position is at least partially offset by the loss on the other. Arbitrage is like hedging in that it is designed to have low risk and it often involves a position in the spot market and an opposite position in the futures market.

iii) An offsetting trade means to simply take an opposite position in the same contract. For example, a trader who buys a gold futures contract can offset the trade by selling a gold futures contract with the same expiration month. This establishes a long and short position in the same contract, which is equivalent to not having a position at all. A cash settlement is permitted at expiration on certain contracts. The settlement price on the last day of trading is automatically equal to the spot price. The account is marked-to-market on the last day and all open positions are automatically closed. If the contract provides for delivery, the

holder of the short position must deliver the commodity to the holder of the long position who pays the futures price on that day, subject to some adjustments provided in certain contracts. Forward contracts are designed to be held to expiration. The terms of the contract are written so as to accommodate delivery if that is the intention of the party. Many forward contracts, however, are cash settled at expiration. If the holder of a forward contract decides to terminate the position early, he would simply re-enter the forward market and request a new offsetting contract. While this is similar to offsetting a futures contract, the forward market may not necessarily have the same liquidity as it did when the contract was opened. While the contract can generally be offset, it may end up being very costly to offset. In addition since both contracts still exist, credit risk remains.

b) i) A centralized trading facility. The exchange is a formal market place for trading the contacts.

ii) Standardized terms. This establishes that certain contracts are identical and, thus, are perfect substitutes for each other.

iii) Rules. The exchange establishes rules and regulations that permit trading to transpire in an orderly manner.

iv) Clearing house. The clearing house associated with the exchange provides a guarantee that each party to the contract will perform as expected. The clearing house also provides the bookkeeping system that keeps tract of the transactions and the margin deposits.

v) Contract development. The exchange continuously monitors economic conditions and develops new contracts designed to meet the changing needs of hedgers and speculators.

c) i) For no-arbitrage condition, we require the risk-less rate r_f satisfy:

$$F = S_0(1 + r_f + C)$$

$$205.2 = 180(1 + r_f + 0.04)$$

$$r_f = 10\%$$

ii) The implied risk-free rate that you earn by buying kryptonite, storing it, and selling it forward at $205.20 per ounce is 10%. If the risk-free borrowing is 5% you should borrow at that rate and invest in hedged kryptonite. If you buy an ounce of kryptonite for $180, you will get $205.20 for it for sure a year from now, minus the storage cost of $180 × 0.04 (=$7.20). If you borrow that $180, you will have to pay principal and interest of $180 × 1.05 (=$89).

Therefore the arbitrage profit is,

$$205.20 - 7.20 - 189 = \$9$$

Paper 3.0(A)
Section B (Theory)
Set 3

Question 1 – Corporate Finance

ZK Plc is a listed company that owns and operates a large number of farms throughout the country. A variety of crops are grown.

Financing structure

The following is an extract from the statement of financial position of ZK Plc at 30 September 2020.

	₦million
Ordinary shares of ₦1 each	200
Reserves	100
9% irredeemable ₦1 preference shares	50
8% loan stock 2021	250
	600

The ordinary shares were quoted at ₦3 per share ex div on 30 September 2020. The beta of ZK Plc's equity shares is 0.8; the annual yield on treasury bills is 5%, and financial markets expect an average annual return of 15% on the market index.

The market price per preference share was ₦0.90 ex div on 30 September 2020. Loan stock interest is paid annually in arrears and is allowable for tax at 2020 30%. The loan stock was priced at ₦100.57 ex interest per ₦100 nominal on 30 September 2020. Loan stock is redeemable on 30 September 2021.
Assume that taxation is payable at the end of the year in which taxable profits arise.

A new project

Difficult trading conditions have caused ZK Plc to decide to convert a number of its farms into camping sites with effect from the 2021 holiday season. Providing the necessary facilities for campers will require major investment, and this will be financed by a new issue of loan stock. The returns on the new campsite business are likely to have a very low correlation with those of the existing farming business.

Required:
a) Using the capital asset pricing model, calculate the required rate of return on equity of ZK Plc at 30 September 2020. Ignore any impact from the new

campsite project. Briefly explain the implications of a beta of less than 1, such as that for ZK Plc. **(4 marks)**

b) Calculate the weighted average cost of capital of ZK Plc at 30 September 2020 (use your calculation in answer to requirement (a) above for the cost of equity). Ignore any impact from the new campsite project. **(6 marks)**

c) Without further calculations, identify and explain the factors that may change ZK Plc's equity beta during the year ending 30 September 2021. **(8 Marks)**

Question 2 – Equity Valuation & Analysis

You have the following information on the divisions of KK Oil:

Division	Asset Beta	Forecast Free Cash Flow(₦mm)	Expected Annual Growth Rate
Oil Exploration	1.4	450	4.0%
Oil Refining	1.1	525	2.5%
Gas & Convenience Stores	0.8	600	3.0%

The risk-free rate of interest is 3% p.a. and the market risk premium is 5% p.a.

a) Compute the overall value of KK Oil (in ₦ millions).
b) Compute the overall asset beta for KK Oil.
c) Compute the overall cost of capital for KK Oil. **(6 marks)**

Question 3 – Equity Valuation & Analysis

You are provided with the following details for Olowo Plc

Table 1: Statement of Financial Position

	2020		2019	
	₦m	₦m	₦m	₦m
Non-Current Asset				
Cost	474.47		409.47	
Depreciation	154.17		90.00	
		320.30		319.47
Current Assets				
Cash	13.00		5.87	
Receivable	30.00		27.00	
Inventory	209.06		189.06	
	252.06		221.93	

Current Liabilities

Payables	(25.05)	(26.05)
Working capital	227.01	195.88
	547.31	515.35
Long-term debt	(240)	(245)
Net Asset	307.31	270.35
Financed by:		
Share capital	160.00	150.00
Retained earnings	147.31	120.35
	307.31	270.35

Table 2: Income Statement, 2020

Revenue	₦300.38
Total operating expenses	(173.74)
Operating profit	127.06
Gain on sales of equipment	4.00
Earnings before interest, taxes, depreciation & amortisation (EBITDA)	131.06
Depreciation and amortization	(71.17)
Earnings before interest & taxes (EBIT)	59.89
Interest	(16.80)
Income tax expense	(12.93)
Net income	₦30.16

Table 3: Supplemental Notes for 2020

Note 1: Olowo had ₦75 million in capital expenditures during the year.

Note 2: A piece of equipment that was originally purchased for ₦10 million was sold for ₦7 million at year-end, when it has a net book value of ₦3 million. Equipment sales are unusual for Olowo.

Note 3: The decrease in long-term debt represents an unscheduled principal repayment; there was no new borrowing during the year.

Note 4: On January 1, 2020, the company received cash issuing 400,000 ordinary shares at a price of ₦25.00 per share.

Note 5: A new appraisal during the year increased the estimated market value of land held for investment by ₦2 million, which was not recognized in 2020 income.

Table 4: Equity Data for 2020 for Olowo Plc

Dividends paid (Millions)	₦3.20
Weighted-average shares outstanding during 2020	16,000,000
Dividend per share	₦0.20
Earnings per share	₦1.89
Beta	1.80

Note: The dividend payout ratio is expected to be constant.

Table 5: Industry and market Data December 31, 2020

Risk-free rate of return	4.00%
Expected rate of return on market index	9.00%
Median industry price/earnings (P/E) ratio	19.90
Expected industry earnings growth-rate	12.00%

a) The portfolio manager of a large mutual fund comments to one of the fund's analyst, Okon: "We have been considering the purchase of Olowo equity shares, so I would like you to analyse the value of the company. To begin, based on Olowo's past performance, you can assume that the company will grow at the same rate as the industry".

   i) Calculate the value of a share of Olowo equity on December 31, 2020, using the Gordon growth model and the capital asset pricing model.

   ii) Calculate the sustainable growth rate of Olowo on December 31, 2020. Use 2020 beginning-of-year values. **(6 marks)**

b) While valuing the equity of Olowo, Okon is considering the use of either cash flow from operations (CFO) or free cash flow to equity (FCFE) in her valuation process.

   i) State two adjustments that Okon should make to cash flow from operations to obtain free cash flow to equity. **(4 marks)**

   ii) Okon decides to calculate Olowo's FCFE for the year 2020, starting with net income. Determine for each of the five supplemental notes given in Table 3 whether an adjustment should be made to net income to calculate Olowo's free cash flow to equity for the year 2020, and the amount of any adjustment.

   iii) Calculate Olowo's free cash flow to equity for the year 2020. **(6 marks)**

c) Okon has revised slightly her estimated earnings growth rate for Olowo and, using normalised (underlying) EPS, which is adjusted for temporary impacts on earnings, now wants to compare the current value of Olowo's equity to that of the

industry, on a growth-adjusted basis. Selected information about Olowo and the industry is given in Table 6.

Compared to the industry, is Olowo's equity overvalued or undervalued on a P/E-to-growth (PEG) basis, using normalised (underlying) earnings per share? Assume that the risk of Olowo is similar to the risk of the industry. **(4 marks)**

(Total: 20 marks)

Table 6: Olowo Vs. Industry

Olowo	
Estimated earnings growth rate	11.0%
Current share price	₦25.00
Normalised (underlying) EPS for 2020	₦1.71
Weighted-average shares outstanding during 2020 16,000,000	
Industry	
Estimate earnings growth rate	12.00%
Median price/earnings (P/E) ratio	19.90

Question 4 – Fixed Income Valuation & Analysis

The following is the list of prices of Zero-coupon bonds of various maturities.

Maturity (Years)	Price (₦)
1	943.40
2	898.47
3	847.62
4	792.16

a) Calculate the YTM of each bond

b) Estimate the implied sequence of forward rate for each year

c) Assuming that the expectation hypothesis is valid, compute the expected price path of the four-year bond as time passes. What is the rate of return of the bond in each year? Show that the expected return equals the forward rate for each year.

d) What should be the current price a 4 - year maturity bond with 8% coupon rate paid annually. If you purchase it at that price, what would your total expected rate of return be over the next year? **(14 marks)**

Question 5 – Fixed Income Valuation & Analysis

You manage the fixed income portfolio of an asset management company based in Abuja. You are considering one of the newly issued 5-year AAA corporate bonds shown in the following table.

Bond	Coupon	Price	Callable	Call Price
A	10%	1,000	Non-callable	N/A
B	10%	1,000	Callable	1,020

a) Suppose that market interest rates decline by 50 basis points (i.e.0.5%). Contrast the effect of this decline on the price of each bond.

b) Which of the two bonds would you prefer when interest rates are expected to rise or to fall? Justify your answer.

c) What would be the effect, if any, of an increase in the volatility of interest rates of the prices of each bond?

d) Describe one advantage and one disadvantage of including callable bonds in a portfolio?   **(8 marks)**

Paper 3.0(A)
Section B (Theory)
Set 3

Solution 1- Corporate Finance

1a) Required return on equity = 5% + 0.8(15% - 5%) = 13%

The beta is a measure of the extent to which historic movements in ZK's share price have correlated with average market returns. A beta of less than 1 means that the share price is less volatile than the market. Thus, at 0.8, it means that if the market index rises by 10% then on average the share price of ZK would be expected to increase by 8%.

This argument does not however mean that the required rate of return on ZK's shares also moves in direct proportion to the required return on the market as this is also affected by the risk free rate.

b) Cost of preference shares = $\dfrac{9}{90} = 10\%$

Total value = ₦50m × 0.9 = ₦45m

Cost of debt

Working with face value of ₦100 nominal

With 1 year to redemption, the following formula can be used to calculate the cost of debt:

$$K_D = \frac{EV}{BV} - 1, \quad \text{where:}$$

EV = total cash flow expected in year 1

= redemption value (₦100) + year 1's interest, net of tax (₦5.60) = ₦105.60

BV = current market value = ₦100.57

∴ K_D = (105.60/100.57) − 1 = 5%

Total value = ₦250m × (100.57/100) = ₦251.43m

Calculation of WACC

Capital	Total value ₦m	Cost %	Hash total ₦m
Equity	600.00	13	78.00
Pref shares	45.00	10	4.50
Loan stock	251.43	5	12.57
	896.42		95.07

WACC = 95.07/896.43 = 10.61%

c) There are three major factors occurring during 2021 which may impact upon the beta of ZK Plc.
   • The opening of a new business venture in campsites;
   • The financing of the new venture with a new issue of bonds;
   • The refinancing of the existing debt which is redeemable in 2021.

The new business venture

The new business venture is significantly different from the existing business. This is indicated by the low correlation of the returns of the two businesses.

The low correlation may diversify the unsystematic risk of the business, but its impact on the beta of the company is uncertain. This will depend on the correlation of the returns on the campsite project with the market portfolio - not their correlation with existing company returns.

Ignoring the impact of debt financing, this new equity beta will be the weighted average of the existing beta and the beta of the new project.

Financing for the new project

The new debt finance will increase financial gearing and thus increase the variability of equity returns on the project and for the company as a whole. If the equity returns become more variable in relation to the market index, then this will increase the equity beta, although the total risk to debt and equity will be unaffected.

Refinancing existing debt

The impact of refinancing on the beta will depend on the type of financing used to redeem the existing debt - if any. If there is like-for-like replacement with new debt, then there will be a minimal impact on the beta, although the terms of the replacement debt instruments may differ.

If however, the debt is redeemed - totally or partially - with new equity then this will reduce gearing, reduce the volatility of equity returns and thus lower the beta.

Other factors

Betas are based on historic returns and may not be stable over time. Past betas are, thus, not necessarily a good guide to the future, as they are affected by random events in relation to the company and the market. Even without the significant operational and financial changes in ZK plc in 2021, the beta would thus be likely to change anyway through normal ongoing events in the farming industry. The direction of change would, however, be in determinant.

Summary

The new beta will be the weighted average of the beta on the existing farming business and the beta of the new leisure business. Both of these may change over time.

Solution 2 – Equity Valuation & Analysis

2a)

Oil Exploration:

$r_{OE} = 0.03 + 1.4(0.05) = 10\%$ $\qquad V_{OE} = \dfrac{FCF}{r-g} = \dfrac{450}{0.1-0.04} = 7,500$

Oil Refining

$r_{OE} = 0.03 + 1.1\,(0.05) = 85\%$ $\qquad V_{OR} = \dfrac{FCF}{r-g} = \dfrac{525}{085-025} = 8,750$

Convenience Stores

$r_{CS} = 0.03 + 0.8\,(0.05) = 70\%$ $\qquad V_{CS} = \dfrac{FCF}{r-g} = \dfrac{600}{07-03} = 15,000$

$V_{KK} = V_{OE} + V_{OR} + V_{CS} = 7,500 + 8,750 + 15,000 = 31,250$

b) $\beta_u = \dfrac{7,500}{31,250}(1.4) + \dfrac{8,750}{31,250}(1.1) + \dfrac{15,000}{31,250}(0.8) = 1.028$

c) $r_u = \dfrac{7,500}{31,250}(0.1) + \dfrac{8,750}{31,250}(0.085) + \dfrac{15,000}{31,250}(0.07) = 8.14\%$

Or

$r_u = 0.03 + 1.028(0.05) = 8.14\%$

Solution 3 – Equity Valuation & Analsis

3a) i) $K_E = r = 4 + 1.8(9-4) = 13\%$

$$P_0 = \frac{D_0(1+g)}{R-g} = \frac{0.20(1.12)}{0.13-0.12} = 22.40$$

ii) g = ROE. b

ROE, as defined in the question is

$$\frac{\text{Net Income}}{\text{Beginning Equity}} = \frac{30.16}{270.35}$$

$$b = \frac{1.89-0.20}{1.89} = \frac{1.69}{1.89}$$

$$g = \frac{30.16}{270.35} \times \frac{1.69}{1.89} = 9.98\%$$

b) i) To obtain free cash flow to equity (FCFE), the two adjustments that Olowo should make to cash flow from operations (CFO) are:

1) Subtract investment in fixed capital: CFO does not take into account the investing activities in long-term assets, particularly plant and equipment. The cash flows corresponding to those necessary expenditures are not available to equity holders and therefore should be subtracted from CFO to obtain FCFE.

2) Add net borrowing: CFO does not take into account the amount of capital supplied to the firm by lenders (e.g., bondholders). The new borrowings, net of debt repayment, are cash flows available to equity holders and should be added to CFO to obtain FCFE.

ii) Note 1: Olowo had ₦75million in capital expenditures during the year.

Adjustment: negative ₦75million

The cash flows required for those capital expenditures (-₦75million) are no longer available to the equity holders and should be subtracted from net income to obtain FCFE.

Note 2: A piece of equipment that was originally purchased for ₦10million was sold for ₦7million at year-end, when it has a net book value of ₦3million. Equipment sales are unusual for Olowo.

Adjustment: positive ₦3million

In calculating FCFE, only cash flow investments in fixed capital should be considered. The ₦7million sale price of equipment is a cash inflow now available to equity holders and should be added to net income. However, the gain over book value that was realised when selling the equipment (₦4 million) is already included in net income. Because the total sale is cash, not just the gain, the ₦3 million net book value must be added to net income. Therefore, the adjustment calculation is: ₦7million in cash received - ₦4 million of gain recorded in net income = ₦3million additional cash received added to net income to obtain FCFE.

Note 3: The decrease in long-term debt represents an unscheduled principal repayment; there was no new borrowing during the year.

Adjustment: negative ₦5million

The unscheduled debt repayment cash flow (- ₦5 million) is an amount no longer available to equity holders and should be subtracted from net income to determine FCFE.

Note 4: On January 1, 2020, the company received cash from issuing 400,000 ordinary shares at a price of ₦25.00 per share.

No Adjustment

Transactions between the firm and its shareholders do not affect FCFE. To calculate FCFE, therefore, no adjustment to net income is required with respect to the issuance of new shares.

Note 5: A new appraisal during the year increased the estimated market value of land held for investment by ₦2million, which was not recognised in 2020 income.

No Adjustment

The increased market value of the land did not generate any cash flow and was not reflected in net income. To calculate FCFE, therefore, no adjustment to net income is required.

iii) Free cash flow to equity (FCFE) is calculated as follows:

FCFE = NI + NCC − ΔFA − ΔWC + Net Borrowing

Where NCC = non-cash charges

ΔFA = investment in fixed capital

ΔWC = investment in working capital

	Million ₦	Explanation
NI =	₦30.16	From Table 2
NCC =	+ ₦67.17	₦71.17 (depreciation and amortisation from Table 2 − ₦4.00)* (gain on sale from Note 2)
ΔFA =	- ₦68.00	- ₦75.00 (capital expenditures from Note 1) + ₦7.00* (cash on sale from Note 2)
ΔWC =	- ₦24.00	- ₦3.00 (increase in accounts receivable from Table 1) + - ₦20.00 (decrease in inventory from Table 1) + - ₦1.00 (decrease in accounts payable from Table 1)
Net Borrowing =	+(- ₦5.00)	- ₦5.00 (decrease in long-term debt from Table 1)
FCFE =	₦0.33	

* Supplemental in Note 2 in Table 3 affects both NCC and ΔFA.

c) Olowo's equity is relatively undervalued compared to the industry on a P/E-to-growth (PEG) basis. Olowo's PEG ratio of 1.33 is below the industry PEG ratio of 1.66. The lower PEG ratio is attractive because it implies that the growth rate at Olowo is available at a relatively lower price than is the case for the industry. The PEG ratios for Olowo and the industry are calculated below:

Olowo

Current price = ₦25.00

Normalised Earnings per share = ₦1.71

Price-to-Earnings Ratio = ₦25/₦1.71 = 14.62

Growth Rate (as a percentage) = 11

PEG Ratio = 14.62/11 = 1.33

Industry

Price-to-Earnings Ratio = 19.90

Growth Rate (as a percentage) = 12

PEG Ratio = 19.90/12 =1.66

Solution 4 – Fixed Income Valuation & Analysis

4 a) For zero coupon bond, the YTM can easily be computed using:

$$\left(\frac{FV}{P_0}\right)^{\frac{1}{n}} - 1$$

1–Year bond: YTM $= \left[\frac{1,000}{943.40}\right]^{\frac{1}{1}} - 1 = 6\%$ $= R_{0,1}$

2 - Year bond: YTM $= \left[\frac{1,000}{898.47}\right]^{\frac{1}{2}} - 1 = 5.5\%$ $= R_{0,2}$

3 - Year bond: YTM $= \left[\frac{1,000}{847.62}\right]^{\frac{1}{3}} - 1 = 5.67\%$ $= R_{0,3}$

4 - Year bond: YTM $= \left[\frac{1,000}{792.16}\right]^{\frac{1}{4}} - 1 = 6\%$ $= R_{0.4}$

b) Implied forward rates

$$F_{0,1} = R_{0,1} = 6\%$$

$$F_{1,2} = \frac{df_1}{df_2} - 1 = \frac{943.40}{898.47} - 1 = 5\%$$

OR $\frac{\left(1+R_{0,2}\right)^2}{\left(1+R_{0,1}\right)^1} - 1 = \frac{(1.055)^2}{1.06} - 1 = 5\%$

$$F_{2,3} = \frac{df_2}{df_3} - 1 = \frac{898.47}{847.62} - 1 = 6\%$$

OR $\frac{\left(1+R_{0,2}\right)^2}{\left(1+R_{0,1}\right)^1} - 1 = \frac{(1.0567)^2}{(1.055)^2} - 1 = 6\%$

$$F_{3,4} = \frac{df_3}{df_4} - 1 = \frac{847.62}{792.16} - 1 = 7\%$$

c) The 3 other forward rates and their calculation are:

$$F_{1,3} = \left[\frac{df_1}{df_3}\right]^{\frac{1}{3-1}} - 1 = \left[\frac{943.40}{847.62}\right]^{\frac{1}{2}} - 1 = 5.5\%$$

OR $\left[\frac{(1+R_{0,3})^3}{(1+R_{0,1})^1}\right]^{\frac{1}{3-1}} - 1 = \left[\frac{(1.0567)^3}{(1.06)^1}\right]^{\frac{1}{3-1}} - 1 = 5.5\%$

$$F_{1,4} = \left[\frac{df_1}{df_4}\right]^{\frac{1}{4-1}} - 1 = \left[\frac{943.40}{792.16}\right]^{\frac{1}{4-1}} - 1 = 6\%$$

OR $\left[\frac{(1+R_{0,4})^4}{(1.06)}\right]^{\frac{1}{4-1}} - 1 = 6\%$

$$F_{2,4} = \left[\frac{df_2}{df_4}\right]^{\frac{1}{4-2}} - 1 = 6.5\%$$

(The alternative method will yield the same result)

d) The expected price path of the 4-years zero coupon bond is as follows:

P_0 $\qquad$ = 792.16

P_1 $\quad \frac{1,000}{(1.06)^3}$ $\quad$ = 839.619

P_2 $\quad \frac{1,000}{(1.065)^2}$ $\quad$ = 881.659

P_3 $\quad \frac{1,000}{1.07}$ $\quad$ = 934.579

The rate of return for each year is now computed.

Year 1: $\frac{P_1+C}{P_0} - 1 = \frac{839.619 + 0}{792.16} - 1 = 6\% = F_{0,1}$

Year 2: $\frac{P_2+C}{P_1} - 1 = \frac{881.619 + 0}{839.619} - 1 = 5\% = F_{1,2}$

Year 3: $\frac{P_3+C}{P_2} - 1 = \frac{934.579 + 0}{881.659} - 1 = 6\% = F_{2,3}$

Year 4: $\frac{FV+C}{P_3} - 1 = \frac{1,000 + 0}{934.579} - 1 = 7\% = F_{3,4}$

Thus, the rate of return for each year equals the forwards rate.

Solution 5 – Fixed Income Valuation & Analysis

5 a) The maturity of each bond is ten years, and we assume annual coupon payment. Since both bonds are selling at par value, the current yield for each bond is equal to its coupon rate.

If the yield declines by 1% to 11%, the BK bond will increase in value to:

$$\frac{120}{1.11} + \frac{120}{(1.11)^2} + \cdots + \frac{1120}{(1.11)^{10}} = 1{,}058.89$$

n =10, 1/y=11, PMT =120, FV=1,000

The price of RT bond will increase, but only to the call price of 1,030. The present value of scheduled payments is greater than 1,030, but the call price puts a ceiling on the actual bond price – the price compression effect.

b) If rates are expected to fall, the BK bond is more attractive: since it is not subject to call, its potential capital gains are greater.

If rates are expected to rise, RT is a relatively better investment. Its higher coupon (which presumably is compensation to investors for the call feature of the bond) will provide a higher rate of return than the BK bond.

c) An increase in the volatility of rates will increase the value of the firm's option to call back the RT bond. If rates go down, the firm can call the bond, which puts a cap on possible capital gains. So, greater volatility makes the option to call back the bond more valuable to the issuer. This makes the bond less attractive to investor.

d) The advantage of a callable bond is the higher coupon (and higher promised yields to maturity) when the bond is issued. If the bond is never called, then an investor earns a higher realized compound yield on a callable bond issued at par than a non-callable bond issued at par on the same date. The disadvantage of the callable bond is the risk of call. If rates fall and bond is called, then the investor receives the call price and then has to reinvest the proceeds at interest rates that are lower than the yield to maturity at which the bond originally was issued. In this event, the firm's savings in interest payments is the investor's loss.

Question 1 – Derivatives & Financial Engineering

1. A non-dividend paying stock is currently priced at ₦20. The risk free rate (continuously compounded) is 3% p.a.

   a) Using the Black-Scholes model, what is the price of a call (European), strike ₦20, maturity is 3 months, assuming the implied volatility is 15%? **(2 marks)**

   b) Using a two-period binomial model, what is the price of this call? **(2 marks)**

   c) In (b) above, what difference would it make in your answer if the option is American rather European? **(2 marks)**

   d) Since you probably found different figures in your answers in (a) and (b), which one would you use as the most correct one and why? **(2 marks)**

   You decide to initiate a short call option: you sell 100 calls (contract size: 100). What should you do in order to delta hedge your position (delta neutral strategy)? Is this hedge protective for any move of the stock? Why? **(2 marks)**

   (10 marks)

Question 2 – Derivatives & Financial Engineering

2. Edo Plc's shares have been trading in a narrow range for the past month, and you are convinced it is going to break far out of that range in the next 3 months. You do not know whether it will go up or down, however. The current share price is ₦100 per share, and the price of a 3-month call option at an exercise price of ₦100 is ₦10.

   Required:

   a) If the risk-free interest rate is 10% per year, continuously compounded, what must be the price of a 3-month put option on the same stock at an exercise price of ₦100? **(5 marks)**

   b) What would be a simple options strategy to exploit your conviction about the stock price's future movements? How far would it have to move in either direction for you to make a profit on your initial investment? **(5 marks)**

   (10 marks)

Question 3 – Portfolio Management

3 a) i) For asset allocation, explain in simple terms the major differences between a top-down and a bottom-up approach.

ii) Explain also the differences between strategic allocation and tactical allocation. Which of the two allocations generates much of the performance on the long run?

b) Mention and describe four different investment constraints that a financial advisor needs to account for while preparing an investment plan for his client. **(10 marks)**

Question 4 – Portfolio Management

4. The following table is a variance/covariance matrix for assets A & B and for the market:

	A	**B**	**Market**
A	0.150	0.030	0.070
B		0.095	0.045
Market			0.060

The expected return of the market is 20% and the risk free interest rate is 4%. Now consider an investor who wants to build a portfolio out of the above two assets with an initial allocation ratio of 60%:40% (60% in A and 40% in B).

a) Calculate the betas of the asset relative to the market respectively.

b) Calculate the beta of the proposed portfolio.

c) Calculate the variance of the portfolio.

d) Calculate the expected returns on Asset A and B. Also what should be the expected returns of the proposed portfolio based on CAPM theory?

e) Calculate the Sharpe ratio of the proposed portfolio and explain if the proposed portfolio is an efficient portfolio. **(18 marks)**

Question 5 – Commodity Trading & Futures

5 a) i) Explain why the futures price converges to the spot price and discuss what would happen if this convergence failed. **(5 marks)**

ii) Explain the function of the settlement committee. Why is the settlement price important in futures markets in a way that the day's final price in the stock market is not so important? **(5 marks)**

b) Consider the following June spot and futures prices for the CBOT silver contract:

Delivery Month	Price ($)
Spot	5.32
August	5.55
October	5.80
December	6.13

i) Assuming no storage costs, compute the implied spot repo rates, using continuous compounding. **(5 marks)**

ii) Assuming no storage costs, compute the implied August forward repo rates, using continuous compounding. **(5 marks)**

(Total= 20 marks)

Question 6 – Commodity Trading & Futures

6. Identify and provide a brief explanation of the factors that affect the spot price of a storable asset. **(2 marks)**

Pilot Solutions - 2021

Paper 3.0(B)
Section B (Theory)
Set 3

Solution 1 – Derivatives & Financial Engineering

1 a)

$$d_1 = \frac{\ln(S/E) + (r + \sigma^2/2)(T)}{\sigma \sqrt{T}}$$

$$d_1 = \frac{\ln(20/20) + (0.03 + 0.15^2/2)(0.25)}{0.15 \sqrt{0.25}} \qquad = 0.1375$$

$$d_2 = d_1 - \sigma \cdot \sqrt{T} = 0.1375 - (0.15)(\sqrt{0.25}) \qquad = 0.0625$$

$$N(d_1) = 0.5547$$

$$N(d_2) = 0.5249$$

$$C = S \cdot N(d_1) - E.e^{-rT} \cdot N(d_2)$$

$$C = (20)(0.5547) - (20)(e^{0.03 \times 0.25})(0.5249) \quad = 0.6744$$

b) The option matures in 3 months, we apply a two-period binomial model. Therefore,

1 period = 1.5 months = 1.5/12   = 0.125 of a year

$$u = e^{\sigma \times \sqrt{T/n}} = e^{0.15 \times \sqrt{0.125}} \qquad\qquad = 1.054464$$

$$d = 1/u \quad = 1/1.054464 \qquad\qquad = 0.948349$$

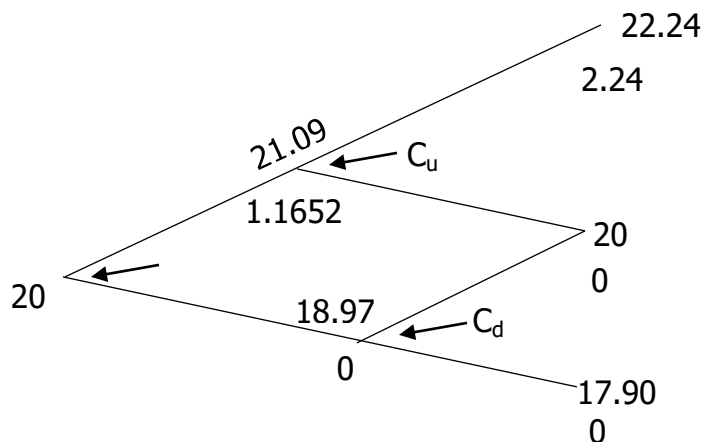$$a = e^{rT/n} = e^{0.03 \times 0.125} \qquad\qquad = 1.003757$$

$$\pi = \frac{a - d}{u - d} = \frac{1.003757 - 0.948349}{1.054464 - 0.948349} \qquad = 0.7649$$

$$1 - \pi = 1 - 0.52215 = 0.47785$$

Discount factor per period is given by

$$e^{-rT/n} = e^{-0.03 \times 0.125} \qquad\qquad = 0.996257$$

The binomial tree is as shown below:

Note: Upper figures = stock prices

Lower figures = option prices

C_u = [(2.24 x 0.5225) + (0 x 0.47785)] x 0.996257 = 1.1652

$C_d = 0$

C_d = [(1.1652 x 0.52215) + (0 x 0.47786)] x 0.996257 = 0.606

c) Since it is never optimal to exercise an American call option on a non-dividend paying stock before the expiration date, the American can has the same price as the European call.

d) The binomial method is not precise enough with a two period model. Using infinite number of periods, the result would strive towards the analytical (B-S) result. The binomial model is only useful when you have a precise scenario during the life path of the option (dividends, change in risk free rate or volatility, etc).

e) Delta of call $= N(d_1) = 0.5547$

100 call contracts = 100 x 100 = 10,000 calls.

Let x = number of stock needed.

The delta of the overall portfolio of options and stock can be detailed as follows:

Security	Qty	Delta/unit	Total delta
Options	-10,000	0.5547	−5,547
Stock	x	1	x
Total portfolio delta			$x - 5,547$

For delta hedging, the portfolio's delta should be zero:

$x - 5547 = 0$

$x = 5547$

Thus, we need to buy 5,547 units of the stock.

The hedge is not perfect because of the convexity of the call option: for a perfect hedge, we would also have to neutralize gamma through a delta-gamma neutral strategy.

Solution 2 – Derivatives & Financial Engineering

2 a)   From put-call parity:

P = C + PV (E) – S

$= 10 + 100e^{-0.1 \times 0.25} - 100 = 7.53$

b)   Purchase a straddle, i.e., both a put and a call on the stock. The total cost of the straddles is: ₦10 + 7.53 = ₦17.53. This is the amount by which the stock would have to move in either direction for the profit on the call or put to cover the investment cost (not including time value of money considerations). Accounting for time value, the stock price would have to move in either direction by:

$(17)(53e^{-0.10 \times 0.25}) = 17.10$

Solution 3 – Portfolio Management

3 a) i) The top-down approach does research and allocation from a high aggregated level (asset classes) down through countries, sectors, etc. to the individual security (firms). By opposition, the bottom-up approach does fundamental analysis on individual assets, and the aggregated result (country, sector, asset classes) is just the consequence of the individual assets selected in the portfolio.

ii) The strategic allocation defines the reference allocation for a portfolio, while the tactical allocation will attempt to fine-tune the strategic allocation given current market conditions. Empirical studies have confirmed that most of the performance is generated at the strategic level.

b)   The following are the different investment constraints.

Time horizon: Investors with long time horizon typically require less liquidity and can afford more risky investment. This low liquidity is needed because funds will not be required before several years. Investors with shorter time horizon will typically favour less risky and more liquid assets because losses are harder to overcome during a short time-period.

Liquidity need: Liquidity can be defined as the speed at which an asset can be converted into cash at a price near market value. Liquidity can be required in order to satisfy near-term goals or cover forthcoming tax obligations etc.

Law and regulation: All investors must respect some laws such as insider prohibition for example. Another example is a fiduciary, who is generally constrained by regulations to make investments decision that are in accordance with the investors wishes.

Taxes: The tax situation of each inventor needs to be investigated thoroughly, due to the high potential impact on returns.

Self-imposed needs and preferences: Some investors are reluctant investing in some securities for various kinds of reasons, e.g. political, ecological, religious, etc. The investor's preferences have to be considered as constraints by the financial adviser.

Operational factors: These are constraints specific to each individual investor. These constraints are at the origin of the sometimes very complex and always different pictures that characterise each client. The financial advisor has to analyse each client's situation individually to assess how the constraints might affect the asset allocation decision.

Solution 4 – Portfolio Management

4) a) The Beta of the assets is given by the following equation:

$$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\text{Var}(R_M)} = \frac{\sigma_{iM}}{\sigma_M^2} \text{, hence } \beta_A = \frac{\sigma_{AM}}{\sigma_M^2} = \frac{0.07}{0.06} = 1.167, \beta_B = \frac{\sigma_{BM}}{\sigma_M^2} = \frac{0.045}{0.06} = 0.75$$

b) The beta of the proposed portfolio is given by:

$$\beta_P = 0.6.\beta_A + 0.4.\beta_B = 1.0$$

c) The variance of the proposed portfolio is calculated as:

$$\sigma_P^2 = (0.6)^2.(0.15) + (0.4)^2.(0.095) + 2.(0.6).(0.4).(0.03) = 0.0836$$

d) Given that $E(R_M) = 20\%, R_F = 4\%$,

the expected returns of the assets according to the CAPM are:

$$E(R_i) = R_F + \beta_i.[E(R_M) - R_F]$$

$$E(R_A) = 4\% + [20\% - 4\%].1.167 = 22.67\%$$

$$E(R_B) = 4\% + [20\% - 4\%].0.75 = 16\%$$

Also since the beta of the proposed portfolio is 1, its expected return is same as that of the expected return of the market portfolio which is given as 20%. This can also be cross checked using the CAPM theory as below:

$$E(R_P) = 4\% + [20\% - 4\%].\,1.0 = 2\%$$

e) The Sharpe ratio of the proposed portfolio is given by the following equation:

$$\text{Sharpe}_p = \frac{E(R_P) - R_F}{\sqrt{\sigma_P^2}} = \frac{20\% - 4\%}{\sqrt{0.0836}} = 0.55$$

The Sharpe of the market is $\text{Sharpe}_M = \dfrac{E(R_M) - R_F}{\sqrt{\sigma_M^2}} = \dfrac{20\% - 4\%}{\sqrt{0.06}} = 0.65$

Now, since the beta of the proposed portfolio is 1, the expected return of the proposed portfolio and that of the market portfolio would be same, but the Sharpe ratio of the market portfolio is higher than that of the proposed portfolio indicating that the proposed portfolio would be dominated by a combination of the market portfolio and the risk free asset. Therefore it is not an efficient portfolio.

Solution 5 – Commodity Trading & Futures

5 a) i) The explanation for convergence at expiration depends on whether the market features delivery or cash settlement, but in each case, convergence depends on similar arbitrage arguments. We consider each type of contract in turn. For a contract with actual delivery, failure of convergence gives rise to an arbitrage opportunity at delivery. The cash price can be either above or below the futures price, if the two are not equal. If the cash price exceeds the futures price, the trader buys the future, accepts delivery, and sells the good in the cash market for the higher price. If the futures price exceeds the cash price, the trader buys the good on the cash market, sells a futures, and delivers the cash good in fulfillment of the futures. To exclude both types of arbitrage simultaneously, the futures price must equal the cash price at expiration. Minor discrepancies can exist, however. These are due to transaction costs and the fact that the short trader owns the options associated with initiating the delivery sequence.

For a contract with cash settlement, failure of convergence also implies arbitrage. Just before delivery, if the futures price exceeds the cash price, a trader can sell the futures, wait for expiration, and the futures price will be set equal to the cash price. This gives a profit equal to the difference between the cash and futures. Alternatively, if the cash price is above the futures price, and expiration is imminent, the trader can buy the futures and wait for its price to be marked up to equal the cash price. Thus, no matter whether the future price is above or below the cash price, a profit opportunity will be available immediately.

In short, the futures and cash price converge at expiration to exclude arbitrage, and failure of convergence implies the existence of arbitrage opportunities.

a) ii) In future markets, the settlement committee determines the settlement price for each contract each day. The settlement price estimates the true value of the contract at the end of the day's trading. In active markets, the settlement price will typically equal the last trade price. In inactive markets, the settlement price is the committee's estimate of the price at which the contract would have traded at the close, if it had traded. The settlement price is important, because it is used to calculate margin requirements and the cash flows associated with daily settlement. In the stock market, there is no practice comparable to daily settlement, so the costing price in the stock market lacks the special significance of the futures settlement price.

b) i) The annualised implied spot repo rate is equal to:

$$C = \frac{1}{1}\ln(F_{0,T}/S_0)$$

August: $C = \frac{12}{2}\ln\left(\frac{5.55}{5.32}\right) = 25.4\%$

October: $C = \frac{12}{4}\ln\left(\frac{5.80}{5.32}\right) = 25.9\%$

December: $C = \frac{12}{6}\ln\left(\frac{6.13}{5.32}\right) = 28.3\%$

ii) The annualised implied forward repo rates are given by:

$$\text{Forward } C = \frac{1}{1}\ln\left(F_{0,T2}/F_{0,T1}\right)$$

October: $C = \frac{12}{2}\ln\left(\frac{5.80}{5.55}\right) = 26.4\%$

December: $C = \frac{12}{4}\ln\left(\frac{6.13}{5.55}\right) = 29.8\%$

Solution 6 – Commodity Trading & Futures

6. The spot price is affected by the cost of carry and the risk premium. The cost of carry is the cost of storing an assets plus the interest foregone by investing funds in the asset. The storage costs include the actual direct physical costs of storage (rent, insurance, security, etc.). The risk premium is the amount by which the expected future price is discounted to compensate the person holding the asset for assuming the risk.