QUESTION & ANSWERS

Examination Paper 3.0B

Derivatives Valuation Analysis
Portfolio Management
Commodity Trading and Futures

Professional Examination
March 2021

Level 2

Question 2 - Derivatives and Financial Engineering

Question 2a

2a1) Show how one can replicate a one-year zero coupon bond with a face value of ₦100 using a share, a put, and a call. (2 marks)

2a2) Suppose that S= ₦100, P= ₦10 and C = ₦15. What must be the one year continuously compounded interest rate? (3 marks)

(Total: 5 marks)

Question 2b

Consider the following information for a call option written on Niger Food Plc (NFP)

Stock price	= ₦96	Delta	= 0.2063
Exercise price	= ₦100	Gamma	= 0.0635
Expiration	= 5 days	Theta	= -48.7155
Volatility (σ)	= 0.4	Vega	= 3.2045
Risk-free rate	= 0.1	Rho	= 0.2643
		Price	= ₦0.5

2b1) Calculate the following for the corresponding put option:

2b1a) Delta

2b1b) Gamma

2b1c) Vega (3 marks)

2b2) If in two days NEP's stock has increased by ₦1 to ₦97, explain what you would expect to happen to the price of the call option. (4 marks)

2b3) An investor owns 240,000 shares of KP Plc that is currently selling for ₦50. A call option on KP is selling for ₦50 with exercise price of ₦48. You are given the following additional information:

$N(d_1) = 0.60$

$N(d_2) = 0.56$

2b3i) Determine the number of call options necessary to create a delta-neutral hedge. (3 marks)

2b3ii.) Assume that the investor now decides to use put option having exercise price of ₦48. What position will be required in put options in order to delta hedge?

(3 marks)

2b3iii.) After implementing the delta-hedging position in a (i) above, KP shares move to ₦51, and consequently call delta becomes 0.62. How will the investors' portfolio of stock and options have to be adjusted to maintain the delta-neutral hedge? (2 marks)

(Total: 20 marks)

Solution to Question 2a

2a1) From the put-call parity, $Ke^{-rT} = S + P - C$

Thus to replicate a 1-year zero coupon bond with a face value of ₦100, buy a share and a European put, and sell a European call, both with exercise price of ₦100.

(2 marks)

2a2) Substituting:

K =100, S =100, P =10

C =15 and T=1

$100e^{-r} = 100 + 10 - 15 = 95$

r = In(100/95) = 5.13%

(3 marks)

~~₦95$e^{0.04}$ = ₦98.88~~

Solution to Question 2b

2b1)

2b1i) Delta of put = delta of call – 1

= 0.2063 – 1 = - 0.7937

2b1ii) Gamma of call = Gamma of put = 0.0635

2b1iii) Vega of call = Vega of put = 3.2045 (3 marks)

2b2) Two variables are changing in this problem, the underlying stock price, S, and the time until expiration, t. Thus, one needs to assess the impact of both DELTA and THETA on the value of the NFP option. DELTA is 0.2063, and THETA is -48.7155. A one naira increase in the price of NFP would be expected to increase the price of the call option by ₦0.2063 = ₦1 × 0.2063. However, as an option contract approaches expiration, the passage of time has a significant adverse effect on the value of the option. Here two days represent 40 percent of the life of the option. The THETA effect is equal to - ₦.2669 = (2/365) × - 48.7155, which is a larger negative effect than the positive impacts of a stock price increase on the value of the option. The combined DELTA and THETA effects are - ₦.0606 = ₦.2063 – ₦2669. Thus, the expected price of the call option is ₦.4394.

(4 marks)

2b3

2b3i) Call delta is $N(d_1) = 0.60$. We do not need $N(d_2)$ to answer any of the questions!

For delta-neutral, the delta of the entire portfolio must equal to zero.

- The delta of each long share is 1. Therefore, the delta of current holding is 240,000 (240,000 shares × 1).
- If the number of call needed is x, total delta of the calls is 0.6x.
- To achieve delta-hedging, the following must hold:

$$240{,}000 + 0.6x = 0$$
$$0.6x = -240{,}000$$
$$x = -240{,}000/06 = -40{,}000$$

($2^{1/2}$ marks)

Because this figure is negative, the investor should sell 400,000 call options. (½ mark)

2b3ii) The delta of each put is 0.6 – 1 = -0.40

If x is the number of put options needed, the total delta of put options needed will be -0.40x. For delta-neutral, the following must hold.

$$240{,}000 - 0.40x = 0$$
$$-0.40x = -240{,}000$$
$$x = -240{,}000/-0.40x$$
$$= 600{,}000 \text{ put options}$$

($2^{1/2}$ marks)

Because the figure is positive, the investor will buy 600,000 put options. (½ mark)

2b3iii) In order to determine the number of call options necessary to maintain the hedge against instantaneous movements in KP's stock prices, we have to calculate the number of calls needed. Let x represent the number of calls needed for delta-hedging. The following must hold:

$$240{,}000 + 0.62x = 0$$
$$x = 240{,}000/0/62 = -387{,}097$$

Currently the investor is short 400,000 calls but this must now be reduced to 387,097 (short). He therefore needs to re-purchase 12,903 calls in order to maintain the hedge.

(2 marks)

Question 3 - Portfolio Management

3a) Given the recent increase of volatility on equity markets, one of your clients is considering the introduction of a portfolio insurance strategy. He is familiar with puts and calls, but not really with portfolio insurance.

3a1) Explain briefly to him what portfolio insurance is. (2½ marks)

3a2) Your client foresees a long period of troubled markets, and asks what would happen if he had to insure his portfolio for several years, knowing that the liquidity on options with a maturity beyond three months is rather poor. Explain to him three rollover strategies, as well as their potential problems. (4½ marks)

3a3) Your client has seen that some banks were offering a Constant Proportion Portfolio Insurance (CPPI) program. However, these programs were not using options. He asks you to explain briefly how these programs work. (2½ marks)

3a4) How would a CPPI strategy perform in a non-directional oscillating market? How about a trending market? (2½ marks)

(12 marks)

3b)

A client wishes to invest in your bank's internal funds that have the following characteristics:

	Expected return	**Volatility (Standard deviation)**	**Correlation coefficient A**	**B**	**C**
Fund A (Money market)	**6%**	**3%**	**1**		
Fund B (Bonds)	**8%**	**5%**	**0.7**	**1**	
Fund C (Equities)	**12%**	**18%**	**0.2**	**-0.3**	**1**

The simple risk-free rate is 5% p.a.

The client is convinced by your bank's funds and decides to invest 10% in fund A, 40% in fund B and 50% in fund C.

3b1) Calculate the expected return and the volatility of the portfolio. (6 marks)

3b2) Explain to your client what is an efficient portfolio and what is an efficient frontier. (3 marks)

3b3) Let's assume that the answers of question a) give a return and a volatility of 9.0%. Demonstrate to your customer, by combining only 2 funds in a new portfolio, that it is possible to find a more efficient portfolio while keeping an expected return of 9.0% (without leverage). (4 marks)

3b4) Eventually, your client thinks that the portfolio he has selected is too risky and wants to have a portfolio with a volatility of 6%. Moreover, he wants to invest only in fund C and in the risk-free rate (R_f = 5%). Find the relative weights of the new portfolio invested in these 2 assets as well as the return of this new portfolio. (3 marks)

(16 marks)

(Total: 28 marks)

Solution to 3a

3a1) Portfolio insurance is a strategy that aims at limiting the downside risk of a portfolio while maintaining its upside potential. Portfolio insurance has traditionally taken two forms: the purchase of put options on the portfolio (static), and the dynamic replication of those same put options (dynamic).

($2^{1/2}$ marks)

3a2) The rollover strategies differ primarily with respect to the choice of the new strike price of the options at the time of the rollover:

- The fixed strike strategy maintain the strike price of the put options constant. It is simple to understand, but might result in requiring deeply in or out of the money put options if the portfolio experiences sharp movements.

($1^{1/2}$ marks)

- The fixed percentage strategy maintains the strike price of the put options equal to a fixed percentage of the underlying portfolio value. If the market keep declining, it does not really offer any downside protection.

($1^{1/2}$ marks)

- The ratchet strategy starts with a strike price equal to a fixed percentage of the underlying portfolio value and can only increase it or keep it constant at the time of a rollover. If the market keeps declining, this strategy will offer some downside protection, but may not be able to participate in a recovery ("cash out").

($1^{1/2}$ marks)

3a3) CPPI is a dynamic asset allocation strategy that essentially buys more equities when they rise and sells them as they decline. To implement a CPPI strategy, the investor selects a floor below which his portfolio value is not allowed to fall. If we think of the difference between the portfolio and floor as a 'cushion' then the CPPI decision rule is to simply keep the exposure to shares a constant multiple of the cushion.

($2^{1/2}$ marks)

3a4) In a flat oscillating market, a CPPI will do relatively poorly, as the investor buys on strength only to see the market weaken, and sells on weakness only to see the market rebound. In a trending bull market, the CPPI strategy will do very well as the investor is buying more shares as they rise. In a trending bear market, the investor will find itself fully in cash at some point.

($2^{1/2}$ marks)

Solution to 3b

3b1) $E(R_p) = 0.1.0.06 + 0.4.0.08 + 0.5.0.12 = 9.8\%$

Variance $= \sigma_p^2 = 0.1^2.3^2 + 0.4^2.5^2 + 0.5^2.18^2 + (2.0.1.0.4.0.7.3.5)$
$+(2.0.1.0.5.0.2.3.18) + (2.0.4.0.5.(-0.3).5.18) = 76.21\ (\%)^2$

Therefore, the volatility is $\sigma R_p = (76.21)^{\frac{1}{2}} = 8.73\%$

(6 marks)

3b2) An efficient portfolio is a portfolio that is located on the efficient frontier. The efficient frontier is the result of the combination of a given set of securities where you only select the most relevant portfolios. In this context you (the client) will choose your optimal portfolio so that it:
- offers a minimum risk for a given level of expected return or
- offers a maximum expected return for a given risk level.

(3 marks)

3b3) Let us choose a combination of 2 funds with a negative correlation coefficient: B and C.

To keep a return of 9% we have:

X_b = weight invested in fund B and $(1 - X_b) = X_c$ = weight invested in fund C

$9 = X_b .8 + (1 - X_b) .12$

$X_b = 0.75$ and $X_c = 1 - 0.75 = 0.25$

(Check: 0.75 .8 + 0.25 .12 = 9% OK)

Now let's calculate the volatility:

$\sigma_p^2 = 075^2.5^2 + 0.25^2.18^2 + 2.0.75.0.25.(-0.3).5.18$
$= 24.188 (\%)^2$

And $\sigma_p = (24.188)^{1/2} = 4.918\%$

We have demonstrated that we were able, by combining only 2 funds, to keep a return of 9% and to decrease the volatility from 9% to 4.918%.

(4 marks)

3b4) For a portfolio made up of a risk-free asset and a risky asset, the variance is given by:

$\text{Variance} = (Wc^2)(\sigma c^2)$, where

W_C = weight of the risky asset in the portfolio

$\sigma_c{}^2$ = variance of the risky asset

In this case:

$6^2 = (Wc^2)(18^2)$

$Wc^2 = \frac{6^2}{18^2}$

$W_c = \frac{1}{3}$

(2 marks)

Thus, $\frac{1}{3}$ of the portfolio value should be invested on fund C and $\frac{2}{3}$ on the risk-free asset. The return is: $E(R_p) = \left(\frac{1}{3} \times 12\right) + \left(\frac{2}{3} \times 5\right) = 7.33\%$

(1 mark)

Question 4- Commodity Trading & Futures

4 a1) A gold futures contract requires the long trader to buy 100 troy ounces of gold. The initial margin requirement is $2,000, and the maintenance margin requirement is $1,500.

4a1i) Tayo goes long one June gold futures contract at the futures price of $320 per troy ounce. When could Tayo receive a maintenance margin call? (2 marks)

4a1ii) Okon sells one August gold futures contract at a futures price of $323 per ounce. When could Okon receive a maintenance margin call? (2 marks)

4a2) A copper futures contract requires the long trader to buy 25,000 Ibs of copper. A trader buys one November copper futures contract at a price of $0.75/Ib. Theoretically, what is the maximum loss this trader could have? Another trader sells one November copper futures contract. Theoretically, what is the maximum loss this trader with a short position could have? (2 marks)

(6 marks)

4b

4bi) Explain what is meant by a perfect hedge. Does a perfect hedge always lead to a better outcome than an imperfect hedge? Explain your answer. (3 marks)

4bii) Give three reasons why the treasurer of a company might not hedge the company's exposure to a particular risk. (3 marks)

4biii) The standard deviation of monthly changes in the spot price of live cattle is (in cents per pound) 1.2. The standard deviation of monthly changes in the futures price of live cattle for the closest contract is 1.4. The correlation between the futures price changes and the spot changes is 0.7. It is now October 15. A beef producer is committed to purchasing 200,000 pounds of live cattle on November 15. The producer wants to use the December live cattle futures contracts to hedge its risk. Each contract is for the delivery of 40,000 pounds of cattle. What strategy should the beef producer follow? ($2^{1/2}$ marks)

4ci) A corn farmer argues "I do not use futures contracts for hedging. My real risk is not the price of corn. It is that my whole crop gets wiped out by the weather". Discuss this viewpoint. Should the farmer estimate his or her expected production of corn and hedge to try to lock in a price for expected production? ($2^{1/2}$ marks)

4cii) An airline executive has argued: "There is no point in our using oil futures. There is just as much chance that the price of oil in the future will be less than the futures price as there is that it will be greater than this price". Discuss the executive's viewpoint. ($2^{1/2}$ marks)

4ciii) It is now July 2020. A mining company has just discovered a small deposit of gold. It will take 6 months to construct the mine. The gold will then be extracted on a more or less continuous basis for 1 year. Futures contracts on gold are available on the New York Commodity Exchange. There are delivery months every 2 months from August 2020 to December 2021. Each contract is for the delivery of 100 ounces. Discuss how the mining company might use futures markets for hedging.($2^{1/2}$ marks)

(16 marks)

(Total: 22 marks)

Solution to Question 4

4a1) The difference between initial and maintenance margin requirements for one gold futures contract is $2,000 – $1,500 =$500. Because one gold futures contract is for 100 troy ounces, the difference between initial and maintenance margin requirements per troy ounce is $500/100, or $5.

(2 marks)

4a1i) Because Tayo has a long position, he would receive a maintenance margin call if the price were to fall below $320 - $5, or $315 per troy ounce.

(2 marks)

4a1ii) Because Okon has a short position, he would receive a maintenance margin call if the price were to rise above $323 + $5, or $328 per troy ounce.

(2 marks)

4a2) Trader with a long position: This trader loses if the price falls. The maximum loss would be incurred if the futures price falls to zero, and this loss would be $0.75/Ib × 25,000 Ibs, or $18,750. Of course, this scenario is only theoretical, not realistic.

Trader with a short position: This trader loses if the price increases. Because there is no limit on the price increase, there is no theoretical upper limit on the loss that the trader with a short position could incur.

(2 marks)

4b)

4bi) A perfect hedge is one that completely eliminates the hedger's risk.

A perfect hedge does not always lead to a better outcome than an imperfect hedge.

It just leads to a more certain outcome.

Consider a company that hedges its exposure to the price of an asset. Suppose the asset's price movement prove to be favorable to the company.

A perfect hedge totally neutralizes the company's gain from these favorable price movements. An imperfect hedge, which only partially neutralizes the gains, might well give a better outcome.

(1 mark for 3)

4bii)

a) If the company's competitors are not hedging, the treasurer might feel that the company will experience less risk if it does not hedge.
b) The shareholders might not want the company to hedge.
c) If there is a loss on the hedge and a gain from the company's exposure to the underlying asset, the treasurer might feel that he or she will have difficulty justifying the hedging to other executives within the organization.

(1 mark for 3)

4biii) The optimal hedge ratio is

$$0.7 \times \frac{1.2}{1.4} = 0.6$$

(½ mark)

The beef producer requires a long position in 200,000 × 0.6 = 120,000 1bs of cattle. The beef producer should therefore take a long position in 3 December contracts closing out the position on November 15.

(2 marks)

4ci) Suppose that the weather is bad and the farmer's production is lower than expected. Other farmers are likely to have been affected similarly. Corn production overall will be low and as a consequence the price of corn will be relatively high. The farmer is likely to be over hedged relative to actual production. The farmer's problems arising from the bad harvest will be made worse by losses on the short futures position. This problem emphasizes the importance of looking at the big picture when hedging. The farmer is correct to question whether hedging price risk while ignoring other risks is a good strategy.

($2^{1/2}$ marks)

4cii) It may well be true that there is just as much chance that the price of oil in the future will be above the futures price as that it will be below the futures price. This means that the use of a futures contract for speculation would be like betting on whether a coin comes up heads or trails. But it might make sense for the airline to use futures for hedging rather than speculation. The futures contract then has the effect of reducing risks. It can be argued that an airline should not expose its shareholders to risks associated with the future price of oil when there are contracts available to hedge the risks.

($2^{1/2}$ marks)

4ciii) The mining company can estimates its production on a month by month basis. It can then short futures contracts to lock in the price received for the gold. For example, if a total of 3,000 ounces are expected to be produced in January 2006 and February 2006, the price received for this production can be hedged by shorting a total of 30 February 2006 contracts. ($2^{1/2}$ marks)