EMPIRICAL EVALUATION OF THE OUT-OF-SAMPLE PERFORMANCE OF ASSET ALLOCATION STRATEGIES IN AN EMERGING MARKET

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ABSTRACT

This study evaluates the out-of-sample performance of an equal-weighted portfolio relative to value-weighted, mean-variance, and Black–Litterman asset allocation strategies in an emerging market, using empirical evidence from Nigeria. Fifteen stocks listed on the Nigerian Exchange Limited as of January 1, 2010, were selected for the analysis. The portfolios were constructed on January 1, 2010, using historical market data, and their out-of-sample performance was evaluated from January 1, 2010, to April 30, 2023. Portfolio performance was assessed using Carhart's four-factor model, as well as the Treynor ratio, Sharpe ratio, Information ratio, M2 measure, and Omega ratio. This study provides a useful guide to retail investors on alternative asset allocation strategies to diversify their portfolios and their relative performance. Our findings indicate that, on a risk-adjusted basis, all the portfolio strategies underperformed the risk-free asset over the sample period, implying that none delivered positive excess returns per unit of risk. The empirical results suggest that while the Value-Weighted (VW) portfolio achieved the highest ordinal rank, there is no statistically significant performance alpha across the various strategies. Despite variations in construction methodology and theoretical complexity, the portfolios yielded comparable returns - a finding that deviates from the prevailing consensus in the literature. Consequently, the study recommends that retail investors in Nigeria may adopt simple portfolio strategies, as they tend to deliver comparable outcomes. However, given the overall negative Sharpe ratio observed, such recommendations should be interpreted with caution. This study is among the first in the Nigerian context to providea comprehensive evaluation of multiple portfolio strategies using a range of portfolio performance measures, including Carhart's four-factor model.

Keywords: Mean-Variance Optimization, Equally-Weighted, 1/N, Value-Weighted, Black-Litterman, Portfolio Construction, Risk-adjusted Performance, Nigeria Exchange Limited

JEL classification: G11, G12, G23, G51

1. INTRODUCTION

Stock market participants, including investors, practitioners, and researchers, have always shown a keen interest in asset allocation strategies as they are central to successful investing. Markowitz (1952) laid the foundation of the mean-variance optimization (MVO) theory, which is the reference point of modern portfolio theory. The essence of the theory is that rational investors will invest in a portfolio that maximizes their returns for a given level of risk. Conversely, investors who are assumed to be risk-averse will select the less risky portfolio for a given level of return. Markowitz also demonstrated that portfolio risk is generally less than the sum of the risks of its constituent securities; investors should therefore be more concerned about the risk of their portfolios than that of separate securities.

Over the years, portfolio managers have applied the principles of the MVO model in constructing portfolios, and researchers have built on it to develop enhanced models, leveraging innovations in fields such as computing technology, behavioral finance, and operational research(Alali &Tolga, 2019).Although the MVO model is theoretically appealing, it has several weaknesses (Lasse, Babu & Levine, 2021; Mynbayeva, Lamb & Zhao, 2022). The model produces unstable and impractical results, largely because the resulting portfolios are extremely sensitive to small changes in the inputs (Best and Grauer, 1991; Black and Litterman, 1992; Michaud, 1989).This problem is accentuated by the fact that the inputs, specifically the mean return and variance-covariance (VCV) matrix, are estimated from historical returns and are noisy estimates of the true parameters (Benninga, 2014).One of the asset allocation strategies that has evolved to address some of the weaknesses of the MVO was developed by Black and Litterman (1991, 1992).The model is flexible, allowing the views of the investor to be incorporated into the optimization process. This view can be expressed in relative or absolute terms and can project an increase or decrease in the returns of assets (Stoilov, Stoilova & Vladimirov, 2021). The resulting weights of assets in the Black-Litterman (BL) portfolio are more rational than the output of the MVO process (Benninga, 2014).

Similarly, DeMiguel, Garlappi & Uppal (2009) presented a piece of interesting evidence of significance in the investment management field with the conclusion from their study that the equally-weighted (EW) portfolio outperformed MVO portfolios in several respects out-of-sample, using sample data from the U.S. Following this, several researchers have corroborated their evidence(for example, Duchin & Levy, 2009; Tu & Zhou, 2009; Pflug, Pitchler&Wozabel, 2012; Bolognesi, Torluccio, &Zuccheri, 2013; Plyakha, Uppal, & Vilkov 2015; and Malladi & Fabozzi, 2017).In addition, Jacobs, Muller, and Weber (2010), using other datasets and asset classes, validated the superior performance of the EW portfolio.

Most of the above-referenced studies, however, were done in the developed markets. In emerging stock markets, there is a huge knowledge gap in this field, and only a few studies have been carried out to evaluate the relative performance of different portfolio strategies. Although Nwakanma & Gbanador (2014), Nnanwa, Urama, & Ezepue (2016), and Gbanador (2018) compared the performance of the EW portfolio with the MVO strategy in the Nigerian stock market, the studies were limited to just two alternative portfolios and a short sample period. This study extends the earlier efforts by evaluating the relative performance of five alternative portfolio strategies, thereby widening the scope. The study period also spans over sixteen years, which increases the statistical power of the analysis. In addition, this study uses Carhart's multiple regression model alongside a range of risk-adjusted performance models that have not been used in the Nigerian context, including the Information ratio, Modigliani-Miller measure, and Omega ratio to provide a robust appraisal.

The research question of this study is, ''Does the EW portfolio outperform the VW portfolio, the Tangency Portfolio with short selling, the Tangency Portfolio without short selling, and the BL optimal active portfolio in the Nigerian stock market? 'Consequently, the objective of the study is to (1) evaluate the performance of the EW portfolio, VW portfolio, Tangency Portfolio with short selling, Tangency Portfolio without short selling, and BL portfolio in the Nigerian stock market, and (2) evaluate whether on a risk-adjusted basis the EW portfolio is superior to the alternative asset allocation strategies in the Nigerian stock market. To undertake this analysis, we selected fifteen of the top twenty stocks on the Nigeria Exchange Limited (NGX) as of January 1, 2010, and collected their historical price data. The historical returns and other relevant market statistics from January 1, 2017, to December 31, 2009, were used to construct the different portfolios of stocks. The performance of the portfolios was evaluated using the return data of the securities from January 1, 2010, to April 30, 2023. We used the statistical analysis of the difference of two means to assess whether the EW strategy is superior to the other asset allocation strategies

This study bridges a knowledge gap by providing a guide to retail investors on alternative portfolio strategies to achieve diversification and grow their wealth in alignment with their risk-return preferences. Further, the output of the study enriches the literature on the performance of various asset allocation strategies in emerging markets generally and particularly in Nigeria. This should be of interest to key stakeholders as both the Nigerian economy and the stock market are rapidly evolving. It is critical that market participants gain access to quality information about the features of the stock markets to guide their investment decisions.

The remaining part of the paper is organized as follows: Section 2 provides the theoretical framework and summary of empirical studies on the performance of asset allocation strategies. This is followed by Section 3, which provides details of the methodology adopted, including market data collected, the sampling approach adopted, and the risk-adjusted models used for performance appraisal. Section 4 presents and discusses the findings of the study in light of the literature. We conclude in Section 5, highlighting our key findings and making policy recommendations.

2. LITERATURE REVIEW

The MVO theory developed by Markowitz (1952) allows investors to construct a portfolio that maximizes their risk-return trade-offs by choosing one of several portfolios located on the efficient frontier. While the theory is theoretically sound and appealing, its use in practice has been challenging, as the tangency portfolio is blighted by various weaknesses (Black &Letterman, 1991). First, the weights, means, and variances of the MVO-based portfolios can be extremely sensitive to the means of underlying securities (Best & Grauer, 1991). This problem is accentuated by the fact that mean returns of assets used in the optimization process are estimated and could be quite unreliable (Markowitz, 1952). Further, the weights of securities in the tangency portfolio are frequently unreasonable and tilted towards a few assets (Black &Litterman, 1991). Hence, over the years, efforts have been made to improve upon the outcome of Markowitz's MVO process. Black &Litterman (1991) developed a model to improve the outcome of the MVO, which has been adjudged effective in constructing portfolios with a more intuitive composition (Benninga, 2014).

The findings of De Miguel *et al*. (2009) havealso motivated more studies on the out-of-sample performance of the MVO portfolios relative to the EW and other heuristic portfolio strategies. DE Miguel *et al.* (2009) compared the performance of different portfolios for different data sets over a period spanning 20 to 50 years. They evaluated the mean-variance portfolio, minimum variance portfolio, Bayes-Stein portfolio, and VW portfolio and found evidence that, overall, the EW portfolio recorded the highest Sharpe ratio. Jacob et. *Al* (2010) evaluated the performance of a large number of asset allocation strategies in the Eurozone using a large sample from January 1973 to December 2008. The study investigated both international diversification in stocks and in asset classes. They found evidence that MVO strategies did not beat simple rule-of-thumb strategies in international stock portfolios. Similarly, the MVO strategies failed to outperform EW allocations in the case of asset allocation strategies.

Overall, the study recommended simple, cost-effective allocation to retail investors as they achieved similar diversification gains as the more sophisticated portfolios. Bolognesi *et al* (2013) appraised the relative performance of the EW and VW portfolios on the DJ Euro Stoxx Index from January 2020 to December 2011. They assessed the performance of the portfolios using monthly, quarterly, semiannual, and annual frequencies. Their result corroborated existing literature that the EW portfolio beat the VW portfolio. They also concluded that the excess returns recorded were not due solely to the size premium. Ernst, Thompson & Miao (2016) demonstrated the superiority of the EW S&P 500 portfolio over Sharpe's VW S&P 500 portfolio in their study. They also investigated the ''MaxMedian rule'', a nonproprietary trading strategy developed to aid retail investors who wish to invest in just 20 stocks in the S&P 500 and self-manage their portfolio. They found that the MaxMedian rule beat the EW portfolio 1.24 times. Edwards, Lazzara & Preston (2018) investigated the sources of the outperformance of EW portfolios. They argue that EW indexing, pragmatic and reasonable market gains historically have been attributed to a few stocks that are difficult to predict. They attribute the outperformance of EW indexes over VW ones to the fact that they have a bias toward smaller companies that tend to outperform large companies, especially during bull markets. In addition, the requirement to rebalance regularly introduces an element of contrarian strategy into the EW index. Taljaard & Eben (2020, 2021) reported that the South African EW portfolio of the Top 40 stocks on the Johannesburg Stock Exchange has underperformed by almost 20% since 2002 relative to the VW portfolio, and sought to provide plausible explanations for the development. The study identified the following factors. First, there is an increasing concentration of market capitalization weights in the Top 40 stocks. Second, the levels of correlations of stocks in the Top 40 have been on the increase, and this lowers diversification benefits and the advantages of frequent rebalancing. Third, the rate of turnover of the index constituents has been high in recent years. The researcher recommended continuing monitoring of the performance of the EW portfolio and a switch to market cap weights when conditions are optimal. They found that this strategy improved both the risk-adjusted returns and the downside risk of the EW portfolio. Plyakha*et al* (2021) sought to explain why EW portfolios beat VW portfolios. First, they suggested that part of the reason was the benefit of size and value effects, which EW portfolios enjoy relative to the VW portfolio. Moreover, they found evidence that regular rebalancing of the EW portfolio was even more important in driving higher total returns. The researchers reported that asset-pricing tests resulted in significantly different inferences depending on whether an EW portfolio or a VW portfolio was used. Yuan & Zhou (2021) investigated the reasons for the historical outperformance of the EW portfolio over Markowitz's MVO portfolio. First, they suggested that estimation error is a major reason why the mean-variance model underperforms, especially when the sample size is small relative to dimensionality. In addition, they show that the EW portfolio delivers an optimal Sharpe ratio in a one-factor model with diversifiable risks as dimensionality increases, notwithstanding the sample size. They concluded that combining the EW strategy with mean-variance estimation when N is small, or combining it with anomalies or machine learning portfolios when N is large, can improve its robustness and outcome. Swade, Nolte, Shackleton & Lohre (2022) investigated the widely reported outperformance of the EW portfolio over the VW ones in their study, using the broad US equity universe and several factor models. They found evidence that EW portfolios have high exposure to the size factor and benefit from the short-term reversal effect, but suffer from negative momentum effect. They suggested that an investor could benefit from the size premium by investing in the equal-weight minus value-weight spread in the US market at a lower cost than the long-short strategy.

Sanford (2019) developed an approach to improve the MVO by building an optimal portfolio that minimizes the expected tail loss of the natural distribution of the Recovery Theorem. He reported that portfolios built using this approach outperformed the equally weighted portfolio and a portfolio formed based on the historical expected tail loss. This version of the MVO portfolio exhibited superior performance, recording the lowest historical tail loss, the smallest maximum drawdown, the highest Sortino ratio, and the highest Sharpe ratio. Huni & Sibindi (2020) appraised the feasibility of constructing mean-variance optimized portfolios on the Johannesburg Stock Exchange (JSE). They reported that the MVO can be an effective strategy for portfolio construction on the JSE. In addition, they demonstrated that the Global minimum variance portfolio derived from the optimization process consistently reduced risk better than the benchmark JSE ALSI, thereby validating the effectiveness of the strategy. Lasse, Babu & Levine (2021) proposed another approach to improve the performance of the mean-variance portfolio through the use of enhanced portfolio optimization. According to the researchers, this model addresses the noise in the estimates of the risk and returns used in the MVO model. They concluded that using this model in both equities and global asset classes resulted in the generation of alpha above the market, the equally weighted portfolio, and the well-known asset pricing factors. Kaczmarek& Perez (2022) applied machine learning tools to build optimized portfolios using Markowitz's MVO and Hierarchical Risk Parity (HRP) optimization models. They applied forest methods to predict expected excess returns and stocks with the highest projected monthly returns from the S&P500 and STOXX600 indexes. They reported that the out-of-sample performance of both the MVO and HRP models beat the equally-weighted portfolio. In a similar study, Chaweewanchon&Chaysiri (2022) used machine learning tools to preselect stocks that are then used as inputs into the MVO process. Using historical data from the Stock Exchange of Thailand 50 Index (SET50) covering January 2015 to December 2020, the study demonstrated that the performance of mean-variance portfolios could be improved substantially in terms of their Sharpe ratio, mean returns, and risk if stocks are preselected using machine learning techniques. Job(2022) compared the performance of the MVO model against several robustified models to identify and recommend the best model to investors in emerging and frontier markets of Africa. A portfolio was constructed from two Emerging Markets and five Frontier Markets indices in Africa. The study reached a similar conclusion to earlier studies that the enhanced models exhibited better characteristics and performance, with superior gross returns, annualized returns, and net portfolio returns over time compared to the basic MVO model. Mynbayeva *et al* (2022) studied Markowitz's MVO model with a view to providing explanations for why it fails in practice. They demonstrated both theoretically and empirically the high probability that the model will not produce the desired result even when the normal distribution is assumed and when an attempt is made to reduce estimation risk using a shrinkage estimator. They developed a method to address the problem by identifying assets that have similar means or variances. Using out-of-sample and bootstrap tests, they found that their method outperformed the naïve Markowitz MVO model and the EW portfolio. They also observed that the bootstrap approach performed better when the means of assets are identical and that covariance shrinkage improves model performance.

Relatively fewer studies have appraised the out-of-sample performance of the BL model over the MVO, EW, and other strategies. Bessler, Opfer & Wolff (2017) developed a version of the BL model designed to overcome the challenge of the MVO and applied it to a multi-asset portfolio comprising global stocks, bonds, and commodity indices, covering from January 1993 to December 2011. They found evidence that the BL portfolio demonstrated superior out-of-sample Sharpe ratio performance over the naïve, mean-variance, minimum-variance, and Bayes-Stein strategies. In addition, the BL portfolio exhibited lower risk, less extreme asset allocation, and more efficient diversification. Harris, Stoja, & Tan (2017) also evaluated the out-of-sample performance of the BL model over the equal-weighted and benchmark market strategy. Using several BL portfolios, they reported that the BL strategy outperformed naïve and benchmark portfolios under a variety of conditions. Allaj (2019) proposed a new approach to applying the BL model in asset allocation and evaluated the comparative out-of-sample performance of the strategy against others. The study concluded that the constrained optimization strategies generally outperformed the unconstrained strategies and the BL-based strategies produced portfolios with superior performance, thereby affirming earlier results by Bessler *et al* (2017) and Haris *et al* (2017). Another study to test the robustness of the Black-Litterman portfolio was done by Meyer-Bullerdiek (2021) who compared the out-of-sample performance of the BL German stock portfolio to the Markowitz MVO portfolio, the DAX (Deutscher Aktienindex- German stock) Index, a reference portfolio, and an EW portfolio. The findings of the study were that the BL portfolio significantly outperformed the DAX, the reference portfolio, and the EW portfolio, although the MV portfolio beat it marginally.

Following the review of the literature, our hypotheses in null form are as follows:

a) The equally-weighted portfolio does not outperform the market value-weighted portfolio on a risk-adjusted basis.

b) The equally-weighted portfolio does not outperform the Black-Litterman portfolio on a risk-adjusted basis.

c) The equally -weighted portfolio does not outperform the Tangency portfolio with short-selling on a risk-adjusted basis.

d) The equally -weighted portfolio does not outperform the Tangency portfolio without short-selling on a risk-adjusted basis.

3. METHODOLOGY

3.1 Stock Market Data

Fifteen of the top twenty stocks by market capitalization on the NGX as of January 1, 2010, were purposively selected into the sample for this study. This sampling approach was adopted based on three key considerations of data availability, liquidity considerations, and market relevance. Only stocks with continuous price data covering both the estimation and evaluation periods were considered suitable. Further, actively traded stocks are liquid, which enhances price discovery and reduces the distortions caused by thin-trading. In addition, large-cap stocks dominate large portfolios and overall market capitalization in Nigeria. For instance, as of March 2026, the NGX 30 accounts for approximately 90% to 95% of the total market capitalization of the NGX All-Share Index (ASI).

However, we do acknowledge that this sampling approach may introduce survivorship bias, as firms that were delisted, suspended, or lacked complete data over the study period were excluded. Furthermore, a concentration on large-cap stocks implies a large-cap bias, which could limit the generalizability of the findings to smaller or less liquid securities. To this end, we admit that the results are most relevant to the performance of the liquid, large-cap segment of the Nigerian equity market rather than the full universe of listed equities.

Table I provides a list of the sample stocks, their ticker symbols, and their market capitalizations as of January 1, 2010.

The weekly close prices of the sample stocks were collected from January 1, 2007, to April 31, 2023, a period of 16 years and 4 months. The required portfolios were created on January 1, 2010, using the return data of the stock from January 1, 2007, to December 31, 2009 (estimation period), and other relevant market statistics. The out-of-sample performance of the portfolios was evaluated over the subsequent period from January 1, 2010, to April 30, 2023.

3.2 Construction of Stock Portfolios

Returns were computed as simple returns (percentage price changes) rather than continuously compounded (log) returns. Further, due to data constraints, the returns reflect capital gains only and do not include dividends or other corporate actions. The limitation of this is that it could underestimate total returns, particularly for dividend-paying stocks. In addition, all the portfolios were constructed at the beginning of the out-of-sample period (January 1, 2010) and held without rebalancing throughout the evaluation period. Portfolio returns were subsequently computed as the weighted sum of constituent equity returns using the initial portfolio weights.

Portfolio 1 is a VW portfolio. The weight of each security in this portfolio is determined by its relative market capitalization to the total market capitalization of the portfolio as illustrated in Equation 1.

$$\mathrm{Wi} = \frac{\mathrm{MCAP_i}}{\sum_{i=1}^{15} \mathrm{MCAP_i}} \tag{1}$$

Portfolio 2 is an EW portfolio (1/N). All the securities in the portfolio have equal weights. Specifically in this study, since we have 15 stocks in our sample, the weight of each security is 1/15, which is 6.67% of the total portfolio weight.

Portfolio 3 is the tangency portfolio with no short selling. In constructing this portfolio, historical price data from the estimation period were converted to return series (Equation 3), from which mean returns (Equation 2) and the variance–covariance (VCV) matrix were derived (Appendix 1) using Equation 4. The tangency portfolio was consequently obtained by maximizing the Sharpe ratio subject to the constraint: wi≥0. The optimization was implemented using the Excel Solver tool, following the methodology outlined in Benninga (2014).

$$\overline{R_\iota} = \frac{1}{N} \sum_{t=1}^{N} R_t \tag{2}$$

$$E(R) = \begin{bmatrix} E(R_1) \\ \vdots \\ E(R_{10}) \end{bmatrix} \tag{3}$$

$$V = \frac{1}{T-1}(R - \bar{R}^{Transpose})^{Transpose}(R - \bar{R}^{Transpose}) \tag{4}$$

Portfolio 4 is a Tangency Portfolio without short-selling restrictions. Its construction follows the same procedure as Portfolio 3 but allows for negative weights, implying that short selling is permitted in the optimization process. It is worthy of note that this assumption is a theoretical optimization concept and does not reflect the short-selling constraints applicable in the Nigerian market.

Portfolio 5 is a Black–Litterman (BL) Portfolio. The BL model was implemented following the framework presented in Benninga (2014), Financial Modeling (4th Edition). The model begins with the derivation of market-implied equilibrium returns, based on market capitalization weights and the VCV matrix (Equations 5, 6, and 7).

$$\mathrm{E(R_P)} = \lambda \mathrm{VP} + \mathrm{R_f} \tag{5}$$

$$\lambda = \frac{\mathrm{E(R_P)} - \mathrm{R_f}}{\sigma_\mathrm{p}} \tag{6}$$

$$\sigma_\mathrm{p} = \mathrm{P^T VP} \tag{7}$$

The investor (analyst) views were formed based on observed performance patterns and earnings forecasts during the estimation period. Two securities- ETI and Oando - were assigned non-zero views, reflecting expected excess returns of 0.02% and 0.01% per week, respectively, above their implied equilibrium returns. These securities were selected as a result of their relative return patterns and perceived potential for outperformance. In practice, the specification of investor views involves an element of subjectivity, which is a key feature of the Black–Litterman model. The procedure is designed to incorporate market-implied views with informed expectations in a systematic and controlled manner. This view was incorporated into the analyst's expected return vector. To get the final expected returns of the portfolio, we took cognizance of the fact that an analyst's view on one or more securities inadvertently reflects on all the securities since they are correlated. To reflect this interdependence among asset returns, a tracking factor matrix was constructed to incorporate the impact of views across all securities based on their covariance structure. It should be noted that this approach is a simplified adaptation of the traditional Black–Litterman framework, as the confidence matrix (Ω) is not explicitly estimated but implicitly incorporated through the adjustment mechanism.

The adjusted expected returns were then used to derive optimal portfolio weights using mean-variance optimization.

$$\theta_i = \frac{\mathrm{Cov}\,(r_i, r_i)}{\mathrm{Var}(r_i)} \tag{8}$$

$$E(R_{adjusted}) = E(R) + \theta\delta \tag{9}$$

$$P = \frac{V^{-1}\,(E(R) - R_f)}{1^T V^{-1}\,(E(R) - R_f)} \tag{10}$$

We minimized the difference between the analyst's expected returns vector and the final adjusted return vector by linear optimization (Equation 9). The adjusted returns are finally used to construct the BL optimized portfolio (Equation 10).

3.3 Risk-Adjusted Performance Measures

The out-of-sample performance of the portfolios was evaluated using a range of risk-adjusted performance evaluation models, including the Treynor ratio, Sharpe ratio, Information ratio, Modigliani-Modigliani (M^2) measure, Omega ratio, and Jensen alpha.

Table II shows the performance evaluation models used and their operational definitions. The Treynor and Sharpe ratios assess the performance of the portfolios from the perspective of their excess returns over the risk-free rate per unit of risk. While the Sharpe ratio uses total risk proxied by the standard deviation of returns, the Treynor ratio uses systematic risk, measured by portfolio beta. The Information ratio measures another dimension of portfolio performance by evaluating the active returns of a portfolio relative to the tracking error, which measures the deviation between the fund's return and index return.

The Modigliani-Modigliani (M^2) measure appraises the returns of a portfolio for the level of risk undertaken relative to the performance of a benchmark. The Omega measure is useful in weighing the chances of the portfolio winning compared to losing by covering all the risk attributes of a portfolio, including the mean, standard deviation, kurtosis, and skewness.

Finally, we applied Carhart's (1997) model, which is an extension of Fama and French's (1993) model. The four-factor model incorporates the momentum effect, which measures the difference between the returns of past winners and past losers and thus the persistence of performance. The systematic risk factors: SMB (size), HML (value), and WML (momentum) were obtained from Kenneth R. French's data library for emerging markets (French Data Library). The factors were constructed using standard portfolio-sorting procedures based on firm size, book-to-market ratios, and past returns. A limitation of this approach is that these factors are not Nigeria-specific and only serve as proxies for systematic risk factors in emerging markets. They may, therefore, not fully capture the Nigerian local market dynamics.

In this study, the NGX All Share Index (NGX ASI) was used as the benchmark, while the return on the Federal Government 2-year bond served as the proxy for the risk-free rate.

Table 1: Performance Evaluation Models and Measurement Approach

	Performance Measure	**Operational Definition**
1	Carhart's Model	$R_t - R_{Ft} = \beta_1 (R_{Mt} - R_{Ft}) + \beta_2 (SMB) + \beta_3 (HML) + \beta_4 (WML) + e$
2	Sharpe Ratio	$\frac{Rp - Rf}{\text{Standard deviation}}$
3	Treynor Ratio	$\frac{Rp - Rf}{\text{Beta}}$
4	Information Ratio	$\frac{Rp - Rb}{\text{Tracking Error}}$
5	Modigliani- Modigliani (M^2)	M^2 = Sharpe Ratio * Std. deviation of benchmark + Rf
6	Omega	$\frac{\Sigma\text{Winning} - \text{Benchmarking}}{\Sigma\text{Benchmarking} - \text{Loosing}}$

**** In Table 2, Rp is the return of the portfolio; Rf is the risk-free rate and Rb is the return of the benchmark*

3.4 Implementation Considerations

The analysis in this study is done on a gross return basis and does not incorporate brokerage fees and bid–ask spreads, rebalancing costs, liquidity constraints, or the impact of regulatory restrictions on short selling. Therefore, this limitation should be factored in when interpreting the results, as it may materially affect actual portfolio performance.

4. RESULT AND DISCUSSION

4.1 Portfolio Composition

Table 2 shows the 5 portfolios constructed and the respective weights of the constituent stocks in each portfolio. It can be observed that the weights of stocks in Portfolios 3 and 4 are rather extreme and lopsided. This is one of the well-documented weaknesses of Markowitz's mean-variance portfolios. Portfolio 3 is concentrated in just 5 stocks, hence is not well diversified. On the other hand, Portfolio 4 has a couple of unreasonable negative weights (representing short selling), in FBNH, GTCO, WAPCO, and UBN. This is not normal in a market that is in equilibrium. Portfolio 5, the BL portfolio, demonstrates an improvement over the MVO strategy. Its components have more reasonable and intuitive weights, and it allows some flexibility by incorporating investors' views into the asset allocation process while using the market-implied returns as a starting point.

Table 2: 5 Portfolios of Stocks with the weight of each Component Security

Stock/ Ticker Symbol	Portfolio 1 (Value Weighted)	Portfolio 2 (Equal-Weighted)	Portfolio 3 (Tangency without short-selling)	Portfolio 4 (Tangency with short selling)	Portfolio 5 (Black - Litterman)
NB	13.60%	6.67%	6.98%	64.2%	10.98%
FBNH	13.32%	6.67%	0.00%	-33.6%	9.76%
ZENITHBANK	12.47%	6.67%	0.00%	6.1%	9.26%
GTCO	10.64%	6.67%	0.00%	-31.6%	9.48%
UBA	8.84%	6.67%	0.00%	-16.6%	7.29%
GUINNESS	6.00%	6.67%	0.00%	10.7%	1.30%
NESTLE	5.62%	6.67%	0.00%	12.1%	7.70%
DANGCEM	6.19%	6.67%	0.00%	34.5%	5.87%
FCMB	4.73%	6.67%	32.75%	52.8%	3.92%
ETI	4.36%	6.67%	24.28%	19.9%	7.14%
WAPCO	3.35%	6.67%	0.00%	-59.6%	6.24%
UNILEVER	3.00%	6.67%	19.41%	47.2%	4.47%
OANDO	2.68%	6.67%	16.59%	23.5%	6.33%
PZ	2.62%	6.67%	0.00%	1.5%	7.43%
UBN	2.59%	6.67%	0.00%	-31.0%	2.83%
	100.00%	**100.00%**	**100.00%**	**100.00%**	**100.00%**

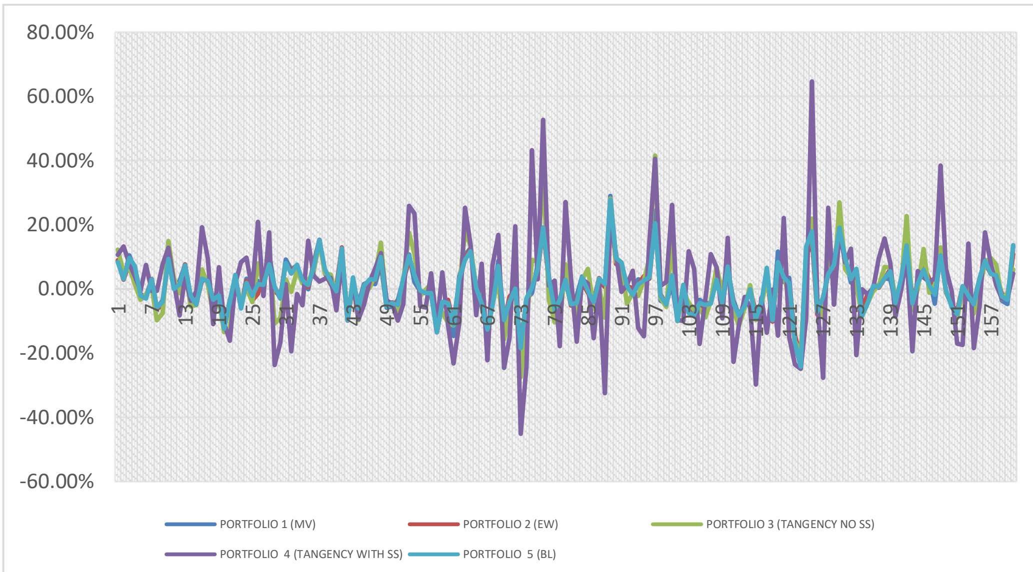
4.2 Raw Risk and Returns Results

Table 3 shows the return and risk measures used as inputs to evaluate the portfolios' risk-adjusted performance. It highlights that the VW portfolio recorded the highest mean return over the study period, followed by the BL portfolio. In terms of risk, the least risky portfolio is the EW portfolio, with a total risk of 0.0050 and a systematic risk of 0.0037, indicating the robust diversification typically associated with EW portfolios. The Tangency Portfolio with short-selling exhibited the highest total and systematic risks at 0.0246 and 0.0048, respectively. This is not surprising given the extreme weights of securities (both positive and negative) in the portfolio and the attendant volatility of returns (Figure I).

Table 3: 5 Portfolios of Stocks with their Return and Risk Measures

	Portfolio 1 (VW)	**Portfolio 2 (EW)**	**Portfolio 3 (Tangency without short-selling)**	**Portfolio 4 (Tangency with short selling)**	**Portfolio 5 (BL)**
Mean Returns	0.8191%	0.7337%	0.6050%	0.7576%	0.7988%
Standard Deviation	0.0760	0.0705	0.0966	0.1569	0.0745
Beta	1.0829	1.0146	1.2081	1.3052	1.0697
Adjusted Beta	1.0556	1.0098	1.1394	1.2045	1.0467
Total Risk	0.0058	0.0050	0.0093	0.0246	0.0056
Systematic Risk	0.0040	0.0037	0.0044	0.0048	0.0039
Idiosyncratic Risk	0.0018	0.0012	0.0049	0.0198	0.0016

Figure I Stream of Returns for 5 Portfolios of Stocks Highlighting the Volatility of Returns



4.3 Discussion of Risk-Adjusted Performance

Table 4 presents the performance of the five portfolios in our study and their relative rankings on each risk-adjusted performance measure. The empirical results show that all five portfolios studied recorded negative Sharpe ratios over the out-of-sample period. This finding has significant implications for the interpretation of portfolio performance and therefore requires careful consideration. Several factors may have accounted for the persistence of negative Sharpe ratios across all portfolios. A key factor is macroeconomic volatility, in particular, persistent inflationary pressures and exchange rate instability, coupled with an attendant high-interest-rate environment during the study period. This raises the benchmark for achieving positive excess returns. In addition, the impact of market inefficiencies, liquidity constraints, information asymmetry, and other structural issues may have further constrained portfolio performance. Further, the relative underperformance of equities compared to fixed-income instruments during parts of the sample period contributed to this outcome. It is particularly noteworthy that the out-of- sample period coincides largely with the post global-economic-meltdown decade in the Nigerian stock market, characterized by massive erosion of investors' confidence and a general downturn in the stock market. The factors highlighted above are a pointer to the fact that even well-diversified or optimized portfolios may struggle to deliver superior risk-adjusted returns in certain macroeconomic environments.

Apart from the Sharpe ratio, which provides a total-risk perspective, other performance measures such as the Treynor ratio, Information ratio, M2 measure, Omega ratio, and Jensen alpha give insights that complement the Sharpe ratio. The Treynor ratio assesses how well portfolios are compensated for systematic risk exposure, while the Information ratio evaluates how closely active returns track the benchmark. The M2 measure translates risk-adjusted performance into percentage return terms, enhancing its interpretation. In addition, the Omega ratio captures other dimensions of return distributions, such as skewness and kurtosis, while Jensen alpha - estimated using the Carhart framework- measures excess returns after controlling for systematic risk factors.

Significantly, the rankings across the measures provide some degree of consistency, indicating [1]a considerable level of reliability. However, their economic significance is hampered by the overall negative risk-adjusted performance recorded across the strategies.

4.4 Absolute vs Relative Performance

The implication of the persistence of a negative Sharpe ratio in the portfolios evaluated is that all the portfolios underperformed the risk-free rate on a risk-adjusted basis, suggesting that an investor would have been better off investing in the risk-free

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asset, proxied by the Federal Government 2-year bond, rather than investing in any of the constructed equity portfolios. However, the primary objective of this study is a comparative evaluation of the portfolios rather than absolute performance optimization. To this end, the rankings across portfolios should be interpreted as measures of relative efficiency, rather than as indicators of overall investment attractiveness.

Table 4: Risk-adjusted Performance of Five Portfolios of Stock and Their Respective Rankings

Risk-Adjusted Measure/Ranking	**Portfolio 1 (VW)**	**Portfolio 2 (EW)**	**Portfolio 3 (Tangency, no short selling)**	**Portfolio 4 (Tangency with short selling)**	**Portfolio 5 (BL)**
Sharpe Ratio	-0.01995	-0.03362	-0.03787	-0.01359	-0.02308
Rank	2	4	5	1	3
Treynor	-0.00144	-0.00235	-0.00321	-0.00177	-0.00164
(Rank)	1	4	5	3	2
Jensen Alpha	0.00033	-0.00061	-0.00167	-0.00003	0.00011
(Rank)	1	4	5	3	2
Information	0.00775	-0.01720	-0.02384	-0.00019	0.00273
(Rank)	1	4	5	3	2
M2	0.00054	-0.00029	-0.00055	0.00092	0.00035
(Rank)	2	4	5	1	3
Omega	0.94848	0.91542	0.90226	0.96365	0.94114
(Rank)	2	4	5	1	3
FINAL RANKINGS	**1st**	**4th**	**5th**	**3rd**	**2nd**

Figure 2: Sharpe Ratio, Treynor Ratio, and Jensen Alpha of the Five Portfolios of Stocks

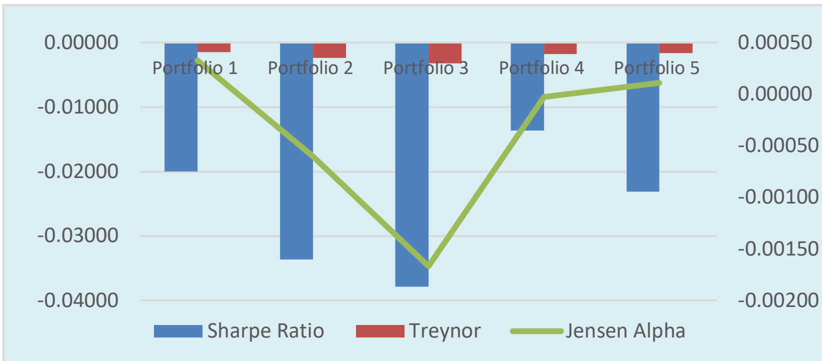

Table 5: Risk-adjusted Performance of Four Alternative Portfolios Using the Jensen Alpha Regression Model with the EW as the Benchmark

	Value-weighted Portfolio (VW)	Tangency no short selling	Tangency with short selling	Black-Litterman Portfolio (BL)
Alpha	0.000367	-0.00156	0.000142	0.000143
P-value	0.9037	0.7550	0.9894	0.9608
Market risk premium (Rm- Rf)	1.0779**	1.2024**	1.3012**	1.0656**
P-value	0.0000	0.0000	0.0000	0.0000

4.5 Interpretation of Portfolio Rankings

Table 5 shows the final rankings of the portfolios based on the average scores from the performance measures. Based on the performance metrics presented in Table 5, the value-weighted (VW) portfolio emerges as the superior strategy. It is followed in descending order of performance by the Black-Litterman portfolio, the Tangency portfolio with short-selling, and the Equal-Weighted (EW) portfolio. The Tangency portfolio, without short-selling, consistently ranks as the least effective strategy. It is important to reiterate, however, that this outperformance is relative rather than absolute. The VW emerged as the least inefficient among the evaluated strategies under the market conditions prevailing during the study period. This distinction is particularly relevant in emerging markets such as Nigeria, where macroeconomic instability, market inefficiencies, and structural constraints may collectively erode overall risk-adjusted returns. It should be emphasised that although the average ordinal ranking approach, used across multiple performance metrics to determine overall portfolio performance, is intuitive and easy to interpret, its weakness is that it is not a formal statistical decision rule and does not test the statistical significance of the observed performance differences. Therefore, the ranking results should not be interpreted as conclusive evidence of portfolio superiority.

From a statistical standpoint, the Jensen Alpha derived from the Cahart's Model (Table 5) presents a more robust conclusion. The empirical results indicate that while the VW, Tangency (with short selling), and BL portfolios yielded marginal positive alphas, and the Tangency (no short selling) portfolio yielded a negative alpha (-0.00156), none of these values achieved statistical significance (all $p > 0.75$). On the other hand, all the strategies demonstrated statistically significant betas (> 1.0), suggesting they exhibited higher systematic risk relative to the EW benchmark. Consequently, the findings suggest that none of the tested portfolio strategies yielded superior risk-adjusted returns, and their performance remains statistically indistinguishable from the benchmark after accounting for market exposure.

Based on these results, we do not reject our hypotheses that state as follows:

a) The equally-weighted portfolio does not outperform the market value-weighted portfolio on a risk-adjusted basis.

b) The equally-weighted portfolio does not outperform the Black-Litterman portfolio on a risk-adjusted basis.

c) The equally -weighted portfolio does not outperform the Tangency portfolio with short-selling on a risk-adjusted basis.

d) The equally -weighted portfolio does not outperform the Tangency portfolio without short-selling on a risk-adjusted basis.

Our findings contradict a large body of work in the literature that reported the superiority of the EW portfolio over the VW portfolio (Plyakha*et al*, 2012; Bolognesi *et al*, 2013; Malladi & Fabozzi, 2017; Yuan & Zhou, 2021). Further, our result is inconsistent with the evidence of Taljaard & Eben (2020, 2021) in South Africa and Meyer-Bullerdiek (2021) in Germany, who reported that the VW portfolio outperformed the EW portfolio in their respective stock markets. Our evidence, therefore, suggests that simpler strategies, such as value-weighted portfolios, may perform comparably with more complex optimization-based strategies in the Nigerian market. However, in view of the negative risk-adjusted returns observed across all portfolios, investors should exercise caution in interpreting these results as actionable investment strategies. While diversification through broad market exposure, such as value-weighted strategies, remains a reasonable approach, this study does not provide sufficient evidence to conclude that such strategies will consistently outperform risk-free assets or alternative investments across all market conditions.

5. CONCLUSION AND POLICY RECOMMENDATION

5.1 Summary

The objective of the study is to evaluate (1) evaluate the performance of the EW portfolio, VW portfolio, Tangency Portfolio with short selling, Tangency Portfolio without short selling, and BL portfolio in the Nigerian stock market, and (2) analyze whether, on a risk-adjusted basis, the EW portfolio is superior to the alternative asset allocation strategies in the Nigerian stock market. The study bridges a knowledge gap by providing a guide to retail investors on alternative portfolio strategies to achieve diversification and grow their wealth in alignment with their risk-return preferences. Further, the output of the study enriches the literature on the performance of various asset allocation strategies in emerging markets generally and particularly in Nigeria. This should be of interest to key stakeholders as both the Nigerian economy and the stock market are rapidly evolving. It is critical that market participants gain access to quality information about the features of the stock markets to guide their investment decisions.

Weekly data on fifteen stocks covering January 1, 2007, to December 31, 2009, were obtained from the NGX to construct five portfolios of stocks using the different asset allocation strategies. The performance of the portfolios was evaluated from January 1, 2010, to April 30, 2023, using the Sharpe ratio, Treynor ratio, Jensen alpha, Information ratio, M^2 measure, Omega ratio, and Carhart's Four Factor regression model. The return of the Federal Government's 2-year bond was used as a proxy for the risk-free rate, while the NGX All Share Index (NGX ASI) was adopted as the benchmark portfolio.

The empirical results, using ordinal rankings, indicate that the VW portfolio relatively outperformed other portfolio strategies; however, there is no evidence of a statistically significant difference in performance across the portfolios. This suggests that, despite differences in construction methods and theoretical sophistication, the strategies ultimately deliver comparable outcomes over the study period. From a practical standpoint, this finding has important implications for retail investors. Given the similarity in performance, investors may be better off adopting simple and cost-effective portfolio strategies, such as equal-weighted or value-weighted approaches, rather than more complex optimization models that may require large amounts of data, expertise, and transaction costs without compensating with superior returns. Furthermore, the analysis reveals that all portfolios underperform the risk-free rate on a risk-adjusted basis. This highlights a fundamental challenge of generating returns that adequately compensate for the level of risk undertaken in the Nigerian market.

5.2 Limitations

While this study offers empirical insights into the out-of-sample performance of asset allocation strategies within an emerging market context, several methodological constraints must be acknowledged. First, the adoption of purposive sampling with a concentration on fifteen large-cap, actively traded stocks on the Nigerian Exchange (NGX) introduces potential survivorship and large-cap biases. This is because delisted firms, merged firms, or those with lower liquidity were excluded. Consequently, the findings may not be fully generalizable to the broader market, particularly to smaller securities, which often exhibit different risk-return profiles. Second, data constraints necessitated the use of price returns alone. This implies that dividends and other corporate actions are excluded from the analysis. Therefore, the study may underestimate total shareholder returns, potentially affecting the relative comparisons between portfolios. Third, the study assumes a static buy-and-hold strategy with no rebalancing. While this simplifies the comparative analysis, it neither fully replicates real-world investment management nor does it account for the impact of transaction costs or liquidity constraints that would inevitably affect actual portfolio performance. Furthermore, the Black–Litterman model was implemented using simplified assumptions regarding investor views and confidence levels (Ω matrix), which may impact the precision of the estimated returns. Similarly, the Carhart four-factor model utilized broad emerging market proxy data rather than Nigeria-specific factors. This may fail to capture the unique Nigerian market dynamics.

5.3 Directions for Future Research

Future studies can build upon the findings of this effort by exploiting several avenues for improvement. Subsequent studies in the field with a focus on the Nigerian market should employ a broader and more representative sample to enhance generalizability and mitigate survivorship bias. The sample should include mid-cap and small-cap stocks, as well as historical data from delisted firms. Further, incorporating total return data (inclusive of dividends and corporate actions) would provide a more comprehensive assessment of the true investment performance of the portfolios. From a practical perspective, future models could incorporate dynamic rebalancing strategies while accounting for transaction costs and liquidity friction. This would align theoretical allocation with the real-world situation. From a methodological perspective, the robustness of the Black–Litterman framework could be improved by explicitly estimating the confidence matrix (Ω matrix). Moreover, the development of Nigeria-specific factor portfolios would enable tailor-made asset pricing analysis. Finally, future researchers should replace heuristic rankings with formal statistical testing methods, such as bootstrap techniques or stochastic dominance analysis, to provide a more robust statistical basis for evaluating performance differences across competing asset allocation strategies.

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Appendices

Appendix 1: The Variance-Covariance Matrix of the Fifteen Stocks Listed on the NGX used to Construct the Mean-Variance and Black-Litterman Portfolios

Variance-Covariance Matrix

	NB Plc	FBN	Zenith	GTB	UBA	Guinn.	Nestle	Dangote	FCMB	ETI	WAP	Unilev.	Oando	PZ	Union
NB Plc	0.0039	0.0023	0.0019	0.0022	0.0018	0.0021	0.0005	0.0003	0.0013	-0.0003	0.0013	0.0022	0.0014	0.0009	0.0014
FBN	0.0023	0.0065	0.0047	0.0035	0.0038	0.0016	0.0004	0.0018	0.0019	0.0002	0.0023	0.0029	0.0013	0.0018	0.0035
Zenith	0.0019	0.0047	0.0081	0.0037	0.0050	0.0017	0.0001	0.0029	0.0028	0.0003	0.0025	0.0035	0.0017	0.0018	0.0043
GTB	0.0022	0.0035	0.0037	0.0067	0.0041	0.0013	0.0015	0.0021	0.0036	0.0007	0.0024	0.0041	0.0012	0.0019	0.0021
UBA	0.0018	0.0038	0.0050	0.0041	0.0093	0.0013	0.0013	0.0026	0.0029	0.0009	0.0029	0.0044	0.0008	0.0026	0.0042
Guinn.	0.0021	0.0016	0.0017	0.0013	0.0013	0.0034	-0.0001	0.0002	0.0011	-0.0003	0.0010	0.0020	0.0004	0.0007	0.0009
Nestle	0.0005	0.0004	0.0001	0.0015	0.0013	-0.0001	0.0039	0.0013	0.0009	0.0003	0.0009	0.0009	0.0008	0.0004	0.0003
Dangote	0.0003	0.0018	0.0029	0.0021	0.0026	0.0002	0.0013	0.0067	0.0022	0.0004	0.0020	0.0019	0.0014	0.0016	0.0017
FCMB	0.0013	0.0019	0.0028	0.0036	0.0029	0.0011	0.0009	0.0022	0.0072	0.0001	0.0022	0.0029	0.0014	0.0016	0.0024
ETI	-0.0003	0.0002	0.0003	0.0007	0.0009	-0.0003	0.0003	0.0004	0.0001	0.0307	0.0006	0.0014	0.0008	0.0012	0.0005
WAPCO	0.0013	0.0023	0.0025	0.0024	0.0029	0.0010	0.0009	0.0020	0.0022	0.0006	0.0052	0.0035	0.0017	0.0018	0.0025
Unilever	0.0022	0.0029	0.0035	0.0041	0.0044	0.0020	0.0009	0.0019	0.0029	0.0014	0.0035	0.0095	0.0018	0.0036	0.0027
Oando	0.0014	0.0013	0.0017	0.0012	0.0008	0.0004	0.0008	0.0014	0.0014	0.0008	0.0017	0.0018	0.0072	0.0019	0.0015
PZ	0.0009	0.0018	0.0018	0.0019	0.0026	0.0007	0.0004	0.0016	0.0016	0.0012	0.0018	0.0036	0.0019	0.0058	0.0021
Union	0.0014	0.0035	0.0043	0.0021	0.0042	0.0009	0.0003	0.0017	0.0024	0.0005	0.0025	0.0027	0.0015	0.0021	0.0092

Appendix 2: The Tracking Factor Matrix Showing the Relative Sensitivity of the Fifteen Listed Stocks in our Sample Used in Computing the Adjusted Returns of Stocks in the Black-Litterman Model

Tracking Factor Matrix

	NB	FBNH	Zenith	GTBank	UBA	Guinness	Nestle	Dangote	FCMB	ETI	WAPCO	Unilever	OANDO
NB Plc	1.0000	0.3565	0.2350	0.3236	0.1886	0.6192	0.1176	0.0382	0.1792	-0.0090	0.2564	0.2297	0.1869
FBNH	0.5984	1.0000	0.5817	0.5248	0.4110	0.4878	0.1091	0.2649	0.2668	0.0059	0.4510	0.3063	0.1770
Zenith	0.4890	0.7212	1.0000	0.5481	0.5366	0.4904	0.0322	0.4271	0.3902	0.0110	0.4848	0.3712	0.2413
GTBank	0.5590	0.5401	0.4550	1.0000	0.4376	0.3935	0.3890	0.3090	0.5010	0.0231	0.4578	0.4285	0.1691
UBA	0.4523	0.5872	0.6183	0.6074	1.0000	0.3994	0.3240	0.3942	0.3999	0.0301	0.5590	0.4683	0.1087
Guiness	0.5351	0.2511	0.2036	0.1968	0.1439	1.0000	-0.0260	0.0301	0.1505	-0.0090	0.1935	0.2087	0.0535
Nestle	0.1169	0.0646	0.0154	0.2238	0.1343	-0.0303	1.0000	0.1946	0.1277	0.0110	0.1740	0.0961	0.1118
Dangote	0.0655	0.2709	0.3523	0.3070	0.2821	0.0599	0.3361	1.0000	0.3088	0.0139	0.3796	0.2017	0.1878
FCMB	0.3298	0.2926	0.3452	0.5339	0.3070	0.3206	0.2365	0.3312	1.0000	0.0041	0.4187	0.3025	0.1902
ETI	-0.0710	0.0276	0.0415	0.1052	0.0988	-0.0828	0.0871	0.0636	0.0173	1.0000	0.1096	0.1521	0.1077
WAPCO	0.3415	0.3578	0.3102	0.3530	0.3105	0.2982	0.2332	0.2946	0.3029	0.0186	1.0000	0.3720	0.2353
Unilever	0.5576	0.4430	0.4330	0.6022	0.4740	0.5863	0.2347	0.2853	0.3988	0.0469	0.6780	1.0000	0.2461
OANDO	0.3474	0.1961	0.2156	0.1820	0.0843	0.1150	0.2092	0.2035	0.1921	0.0254	0.3284	0.1884	1.0000
PZ	0.2373	0.2726	0.2167	0.2881	0.2733	0.2023	0.1042	0.2384	0.2210	0.0377	0.3384	0.3818	0.2597
Union	0.3661	0.5328	0.5316	0.3086	0.4508	0.2750	0.0691	0.2589	0.3286	0.0166	0.4812	0.2809	0.2112